Mr. Wm Eaton

[illegible]

THE SCHOLAR'S ARITHMETIC,

OR

Federal Accountant.

CONTAINING,

I. Common Arithmetic, *the Rules and Illustrations*.

II. Examples and Answers with Blank Spaces, sufficient for their operations by the SCHOLAR.

III. To each Rule a *Supplement*, comprehending, 1. QUESTIONS on the nature of the Rule, its use, and the manner of its operations. 2. EXERCISES.

IV. Federal Money, with rules for all the various operations in it—to reduce Federal to Old Lawful, and Old Lawful to Federal Money.

V. Interest cast in Federal Money, with Compound Multiplication, Compound Division and Practice, wrought in Old Lawful and in Federal Money, *the same questions being put, in separate columns on the same page, in each kind of money, by which these two modes of account become contrasted, and the great advantage gained by reckoning in Federal Money easily discerned.*

VI. Demonstrations, *by engravings, of the reason and nature of the various steps in the extraction of the Square and Cube Roots,* not to be found in any other treatise on Arithmetic.

VII. *Forms of Notes, Deeds, Bonds, and other instruments of writing.*

THE WHOLE IN A FORM AND METHOD ALTOGETHER NEW, FOR THE EASE OF THE MASTER AND THE GREATER PROGRESS OF THE SCHOLAR.

BY DANIEL ADAMS, M. B.

SEVENTH EDITION.

Montpelier, Vt.—PRINTED BY WRIGHT & SIBLEY, for *JOHN PRENTISS*—[Proprietor of the copy-right,]

and sold at his book-store, wholesale and retail.—sold also by the principal book-sellers in New-England.

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Price 10 Dollars per doz. 1 Dollar single.

RECOMMENDATIONS.

New-Salem, Sept. 14*th*, 1801.

HAVING attentively examined "*The Scholar's Arithmetic*," I cheerfully give it as my opinion, that it is well calculated for the instruction of youth; and that it will abridge much of the time now necessary to be spent in the communication and attainment of such Arithmetical knowledge, as is proper for the discharge of business.

WARREN PIERCE.

Preceptor of New-Salem Academy.

Groton Academy, Sept. 2, 1801.

Sir,

I HAVE perused with attention "*The Scholar's Arithmetic*," which you transmitted to me some time since. It is, in my opinion, better calculated to lead students in our Schools and Academies into a complete knowledge of all that is useful in that branch of literature, than any other work of the kind I have seen. With great sincerity I wish you success in your exertions for the promotion of useful learning; and I am confident, that to be generally approved, your work needs on[ly] to be generally known.

WILLIAM M. RICHARDSO[N]

Preceptor of the Acade[my].

EXTRACT

Of a letter from the Hon. JOHN WHEELOCK, *L. L. D. Pr[esi]dent of Dartmouth College, to the* AUTHOR.

"THE Scholar's Arithmetic is an improvement on forme[r] productions of the same nature. Its distinctive order and s[up]plement will help the learner in his progress; the part on F[ed]eral Money makes it more useful; and I have no doubt [that] the whole will be a new fund of profit in our country."

Sept. 7*th*, 180[1].

THE Scholar's Arithmetic contains most of the import[ant] Rules of the Art, and something, also, of the curious and [en]tertaining kind.

The subjects are handled in a simple and concise manner.

While the Questions are few, they exhibit a considerable variety. While they are, generally, easy, some of them afford scope for the exercise of the Scholar's Judgment.

It is a good quality of the Book, that it has so much to do with Federal Money.

The plan of showing the reasons of the operations in the extraction of the Square and Cube Roots is good.

DANIEL HARDY, Jun.
Preceptor of Chesterfield Academy.

Extract of a letter from the Rev. LABAN AINSWORTH, *of Jaffrey, to the publisher of the 4th Edition, dated Aug.* 3, 1807.

"THE Superiority of the Scholar's Arithmetic to any book of the kind in my knowledge, clearly appears from its good effect in the Schools I annually visit. Previous to its introduction, Arithmetic was learned and performed mechanically; since, scholars are able to give a rational account of the several operations in Arithmetic, which is the best proof of their having learned to good purpose."

PREFACE [TO THE 2d EDITION] DEDICATORY

TO

SCHOOL MASTERS.

GENTLEMEN,

AFTER expressing my sincere thanks for your kind and very ready acceptance of the first Edition of the SCHOLAR'S ARITHMETIC, permit me now to offer for your further consideration and favor, the SECOND EDITION, which, with its CORRECTIONS and ADDITIONS, it is hoped, will be found still more deserving of your approbation.

The testimony of many respectable Teachers has inspired a confidence to believe, that this work, where it has been introduced into Schools, has proved a kind assistant towards a more speedy and thorough improvement of Scholars in Numbers, and at the same time, has relieved masters of a heavy burden of writing out Rules and Questions, under which they have so long laboured, to the manifest neglect of other parts of their Schools.

To answer the several intentions of this work, it will be necessary that it should be put into the hands of every Arithmetician; the blank after each example is designed for the operation by the Scholar, which being first wrought upon a slate, or waste paper, he may afterwards transcribe into his book.

The SUPPLEMENT to each Rule in this work is a novelty. I have often seen books with questions and answers, but in my humble opinion, it is no evidence that the Scholar comprehends the principles of that science which is his study, because that he may be able to repeat, *verbatim* from his book, the answer to a question on which his attention has been exercised, two or three hours, to commit to memory. Study is of but little advantage to the human mind without reflection. To force the Scholar into reflections of his own, is the object of those *Questions* unanswered, at the beginning of each Supplement. The *Exercises* are designed, tests of his judgment. The Supplements may be omitted the first time going through the book, if thought proper, and taken up afterwards as a review.

Through the whole it has been my greatest care to make myself intelligible to the Scholar; such rules and remarks as have been compiled from other authors are included in quotations; the *Examples*, many of them, are extracted. This I have not hesitated to do, when I found them suited to my purpose.

Demonstrations of the reason and nature of the operations in the extraction of the Square and Cube Roots have never been attempted, in any work of the kind before, to my knowledge. It is hoped these will be found satisfactory.

I have only to add, that any intimation of amendments or defects by the candid and experienced of your order, will be thankfully received by

Gentlemen,

Your most humble, and

most obedient servant,

DANIEL ADAMS.

Leominster, Mass. *Oct.* 1, 1802.

SEVENTH EDITION.

The Seventh edition is printed, page for page, from the Sixth.

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$=$ The sign of equality; as 100 *cts*=1 *Doll.* signifies that 100 cents are *equal* to 1 dollar.

$+$ Saint George's Cross, the sign of addition; as $2+4=6$, that is 2 added to 4 is equal to 6.

$-$ The sign of subtraction; as $6-2=4$; that is, 2 taken from 6 leaves 4.

$\times$ Saint Andrew's Cross, the sign of multiplication; as $4\times6=24$; that is, 4 times 6 is equal to 24.

$\div$ *or*)(Reversed Parenthesis, the sign of division; as 3)6(2, that is, 6 divided by 3 is equal to 2, *or* $6\div3=2$.

: : : : The sign of proportion; as, 2 : 4 : : 8 : 16, that is, as 2 is to 4 so is 8 to 16.

THE SCHOLAR'S ARITHMETIC.

INTRODUCTION.

ARITHMETIC is the art or ſcience which treats of numbers.

It is of two kinds, *theoretical* and *practical.*

The THEORY of Arithmetic explains the nature and quality of numbers, and demonſtrates the reaſon of practical operations. Conſidered in this ſenſe Arithmetic is a *Science.*

PRACTICAL ARITHMETIC ſhews the method of working by numbers, ſo as to be moſt uſeful and expeditious for buſineſs. In this ſenſe Arithmetic is an *Art.*

DIRECTIONS TO THE SCHOLAR.

Deeply impreſs your mind with a ſenſe of the importance of arithmetical knowledge. The great concerns of life can in no way be conducted without it. Do not, therefore, think any pains too great to be beſtowed for ſo noble an end. Drive far from you idleneſs and ſloth; they are great enemies to improvement. Remember that youth, like the morning, will ſoon be paſt, and that opportunities once neglected, can never be regained. Firſt of all things, there muſt be implanted in your mind a fixed delight in ſtudy; make it your inclination; "A deſire accompliſhed is ſweet to the ſoul." Be not in a hurry to get through your book too ſoon. Much inſtruction may be given in theſe few words, UNDERSTAND EVERY THING AS YOU GO ALONG. Each rule is firſt to be committed to memory; afterwards, the examples in illuſtration, and every remark, are to be peruſed with care. There is not a word inſerted in this Treatiſe, but with a deſign that it ſhould be ſtudied by the Scholar. As much as is poſſible, endeavor to do every thing of yourſelf; *one* thing found out by your own thought and reflection, will be of more real uſe to you, than *twenty* things told you by an Inſtructor. Be not overcome by little *ſeeming* difficulties, but rather ſtrive to overcome *ſuch* by patience and application; ſo ſhall your progreſs be eaſy, and the object of your endeavors ſure.

On entering upon this moſt uſeful ſtudy, the firſt thing which the Scholar has to regard is

NOTATION.

NOTATION is the art of expreſſing numbers by certain characters or figures; of which there are two methods. 1. The *Roman method,* by Letters. 2. The *Arabic method,* by figures. The latter is that of general uſe.

In the Arabic method all numbers are expreſſed by theſe ten characters or figures.

1	2	3	4	5	6	7	8	9	0
Unit, or one;	two;	three;	four;	five;	ſix;	ſeven;	eight;	nine;	cypher, or nothing.

The nine firſt are called *ſignificant figures*, or *digits*, each of which ſtanding by itſelf or alone, invariably expreſſes a particular and certain number; thus, 1 ſignifies *one*, 2 ſignifies *two*, 3 ſignifies *three*, and ſo of the reſt until you come to *nine*, but for any number more than *nine*, it will always require two or more of thoſe figures ſet together in order to expreſs that number.

This will be more particularly taught by

NUMERATION.

Numeration teaches how to *read* or *write* any ſum or number by figures.

In ſetting down numbers for arithmetical operations, eſpecially with beginners, it is uſual to begin at the *right hand*, and proceed towards the *left*.

Example. If you wiſh to write the ſum or number 537, begin by ſetting down the *ſeven*, or right hand figure, thus, 7, next ſet down the *three*, at the left hand of the ſeven, thus, 37, and laſtly the *five*, at the left hand of the three, thus 537, which is the number propoſed to be written.

In this ſum thus written you are next to obſerve, that there are *three places*, meaning the ſituations of the three different figures, and that each of theſe places has an appropriated name. The *firſt place*, or that of the right hand figure, or the place of the 7, is called *Unit's place*; the *ſecond place*, or that of the figure ſtanding next to the right hand figure, in this caſe the place of the 3, is called *ten's place*; the *third place*, or next towards the left hand, or place of the 5, is called *hundred's place*; the next, or *fourth place*, for we may ſuppoſe more figures to be connected, is *thouſand's place*; the next to this, *tens of thouſand's place*, and ſo on to what length we pleaſe, there being particular names for each place. Now every figure ſignifies differently, accordingly as it may happen to occupy one or the other of theſe places.

The value of the firſt or right hand figure, or of the figure ſtanding in the place of *units*, in any ſum or number, is juſt what the figure expreſſes ſtanding alone or by itſelf; but every other figure in the ſum or number, or thoſe to the left hand of the firſt figure, have a different ſignification from their true or natural meaning, for the next figure from the right hand towards the left, or that figure in the place of *tens* expreſſes ſo many times ten, as the ſame figure ſignifies units or ones when ſtanding alone, that is, it is *ten times* its ſimple, primitive value; and ſo on, every removal from the right hand figure making the figure thus removed *ten times* the value of the ſame figure when ſtanding in the place immediately preceding it.

Hund. Tens. Units.

Example. Take the ſum 3 3 3, made by the ſame figure three times repeated. The firſt or right hand figure, or the figure in the place of *units*, has its natural meaning or the ſame meaning as if ſtanding alone, and ſignifies *three* units or ones; but the ſame figure again towards the left hand in the ſecond place, or place of tens, ſignifies not three units, but *three tens*, that is, *thirty*, its value being increaſed in a *tenfold proportion*; proceeding on ſtill further towards the left hand, the next figure, or that in the third place, or place of *hundreds*, ſignifies neither *three* nor *thirty*, but *three hundred*, which is ten times the value of that figure, in the place immediately preceding it, or that in the place of *tens*. So you might proceed and add the figure 3, fifty or an hundred times, and every time the figure was added, it would ſignify *ten times* more than it did the laſt time before.

A CYPHER standing alone is of no signification, yet placed at the right hand of another figure it increases the value of that figure in the same ten fold proportion, as if it had been preceded by any other figure. Thus 3, standing alone, signifies *three*; place a cypher before it, (30) and it no longer signifies *three* but *thirty*; and another cypher (300) and it signifies three hundred.

The value of figures in conjunction, and how to read any sum or number, agreeably to the foregoing observations, may be fully understood by the following

TABLE.

Billions.	Hund. of Thous. of Millions.	Tens of Thous. of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
2	1	6	7	2	3	5	4	2	1	8	3	6
	3	4	0	7	6	2	1	4	6	3	1	2
		1	3	0	2	5	0	3	7	6	4	5
			4	1	3	9	8	2	1	0	6	4
				2	7	0	2	1	3	6	7	5
					4	6	3	2	7	8	9	1
						1	2	3	4	6	3	2
							2	3	4	5	6	7
								8	9	0	9	8
									7	6	5	4
										1	2	3
											4	5
												7

The words at the head of the Table shew the signification of the figures against which they stand; and the figures shew how many of that signification are meant. Thus, *units* in the first place signifies *ones*, and 6 standing against it shew that *six ones*, or individuals are here meant; *tens* in the second place shew that every figure in this place means so many *tens*, and 3 standing against it shews that *three tens* are here meant, equal to *thirty*, what the figure really signifies. *Hundreds* in the third place shew the meaning of figures in this place to be *Hundreds*, and 8 shews that *eight hundreds* are meant. In the same manner the value of each of the remaining figures in the Table is known. Having proceeded thro' in this way, the sum of the first line of figures or those immediately against the words, will be found to be *Two billions, one hundred sixty seven thousands, two hundred and thirty five millions; four hundred twenty one thousands; eight hundred and thirty six*. In like manner may be read all the remaining numbers in the Table.

Those words at the head of the Table are applicable to any sum or number, and must be committed perfectly to memory so as to be readily applied on any occasion.

For the greater ease in reckoning, it is convenient and often practised in public offices and by men of business, to divide any number into periods and half periods, as in the following manner:

TRILLIONS.	Hundred thousand of billions	Ten thousand of billions	Thousand billions	Hundred billions	Ten billions	BILLIONS.	Hundred thousand of millions	Ten thousand millions	Thousand millions	Hundred millions	Ten millions	MILLIONS.	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
5.	3	7	9,	6	3	4.	5	2	1,	7	6	8.	5	3	2,	4	6	7

The first six figures from the right hand are called the *unit period*, the next six the *million period*, after which the *trillion*, *quadrillion*, *quintillion*, periods, &c. follow in their order.

Thus, by the use of ten figures may be reckoned every thing which can be numbered; things, the multitude of which far exceed the comprehension of man.

"It may not be amiss to illustrate by a few examples the extent of numbers, which are frequently named without being attended to. If a person employed in telling money, reckon an hundred pieces in a minute, and continue at work ten hours each day, he will take seventeen days to reckon a million; a thousand men would take 45 years to reckon a billion. If we suppose the whole earth to be as well peopled as Britain, and to have been so from the creation, and that the whole race of mankind had constantly spent their time in telling from a heap consisting of a quadrillion of pieces, they would hardly have yet reckoned a thousandth part of that quantity."

After having been able to read correctly to his instructor all the numbers in the foregoing Table, the learner may proceed to write the following numbers out in words.

6

98

437

6012

72845

149703

9783016

53721680

SECTION I.

FUNDAMENTAL RULES OF ARITHMETIC.

THESE are four, Addition, Subtraction, Multiplication, and Division ; they may be either *simple* or *compound ;* simple, when the numbers are all of one sort or denomination ; compound, when the numbers are of different denominations.

They are called, *Principal* or *Fundamental Rules*, because that all other rules and operations in Arithmetic are nothing more than various uses and repetitions of these four rules.

The object of every arithmetical operation, is, by certain given quantities which are known, to find out others which are unknown. This cannot be done but by changes effected on the given numbers ; and as the only way in which numbers can be changed is either by increasing or by diminishing their quantities, and as there can be no increase or diminution of numbers but by one or the other of the above operations, it consequently follows, that *these four rules* embrace the whole art of Arithmetic.

§ 1. Simple Addition.

SIMPLE ADDITION is the putting together of two or more numbers, of the same denomination, so as to make them one whole or total number ; as 3 dollars, 6 dollars, and 8 dollars added or put together, make 17 dollars.

RULE.

" Write the numbers to be added one under another, with units under " units, tens under tens, and so on. Draw a line under the lower number, " then add the right hand column ; and if the sum be under *ten*, write it at " the foot of the column ; but if it be *ten*, or an *exact number* of tens, *write a cy-* "*pher ;* and if it be not an exact number of tens, write the *excess* above tens at " the foot of the column ; and for *every ten* the sum contains, *carry one* to the " next column, and add it in the same manner as the former. Proceed in " like manner to add the other columns, carrying for the tens of each to the " next, and mark down the full sum of the left hand column."

PROOF.

Reckon the figures from the top downwards, and if the work be right, this amount will be equal to the first ;—or, what is often practised, " cut off " the upper line of figures and find the amount of the rest ; then if the a- " mount and upper line, when added, be equal to the sum total, the work is " supposed to be right."

EXAMPLES.

	Thous.	Hund.	Tens.	Units.		Thous.	Hund.	Tens.	Units.		Hund.	Tens.	Units.		Uts.	
1. What will be the amount of	3	6	1	2	dollars ;	8	0	4	3	dollars ;	6	5	1	dollars, and of	3	dollars, when added together ?

Here are four sums given for addition ; two of them contains *units, tens, hundreds, thousands ;* another of them contains *units, tens, hundreds ;* and a fourth contains *units* only. The first step to prepare these sums for the operation of addition, is to write them down, units under units, tens under tens, and so on as in the following manner.

	T. of Thou.	Thousands.	Hundreds.	Tens.	Units.	
The four given sums for addition placed as the rule directs.		3	6	1	2	dollars.
		8	0	4	3	dollars.
			6	5	1	dollars.
					3	dollars.
Answer, or amount,	1	2	3	0	9	dollars.
Amount of the three lower lines,		8	6	9	7	
Proof,	1	2	3	0	9	

To find the answer or amount of the sums given to be added, begin with the right hand column, and say 3 to 1 is 4, and 3 is 7, and 2 is 9 ; which sum (9) being less than *ten*, set down directly under the column you added. Then proceeding to the next column, say again, 5 to 4 is 9, and 1 is 10, being *even ten*, set down 0, and carry 1 to the next column, saying 1, which I carry to six is 7, and 0 is nothing, but 6 is 13 ; which sum (13) is an excess of 3 over even tens ; therefore set down 3 and carry 1 for the 10 to 8 in the next column, saying 1 to 8 is 9, and 3 is 12 ; this being the last column, set down the whole number, (12) placing the 2, or unit figure directly under the column, and carrying the other figure, or the 1, forward to the next place on the left hand, or to that of *tens of thousands*, and the work is done.

It may now be required to know if the whole be right. To exhibit the method of proof let the upper line of figures be cut off as seen in the example. Then adding the three lower lines which remain, place the amount (8697) under the amount first obtained by the addition of all the sums, observing carefully that each figure falls directly under the column which produced it ; then add this last amount to the upper line which you cut off ; thus, 7 to 2 is 9 ; 9 to 1 is 10 ; carry 1 to 6 is 7 and 6 is 13 ; 1 which I car-

ry again to 8 is 9 and 3 is 12, all which being set down in their proper places, as seen in the example, compare the amount (12309) last obtained, with the firſt amount (12309) and if they agree, as it is ſeen in this caſe they do, then the work is judged to be right.

Note. The reaſon of *carrying for ten* in all ſimple numbers is evident from what has been taught in Notation. It is becauſe 10 in an inferior column is juſt equal in value to 1 in a ſuperior column. As if a man ſhould be holding in his *right hand* half piſtareens, and in his left hand, dollars. If you ſhould take 10 half piſtareens from his right hand, and put one dollar into his left hand, you would not rob the man of any of his money, becauſe 1 of thoſe pieces in his left hand is juſt equal in value to 10 of thoſe in his right hand.

2. Add together { 37652 guineas, 21304 guineas, 90163 guineas, 25321 guineas } ſo as to find the whole number of guineas.

3 7 6 5 2	guineas
2 1 3 0 4	guineas
9 0 1 6 3	guineas
2 5 3 2 1	guineas
1 7 4 4 4 0	whole number of guineas.
1 3 6 7 8 8	
1 7 4 4 4 0	Proof.

The Scholar who has given proper attention to his rule, and the foregoing examples, will of himſelf be able to work the following; always remembering to carry one for every 10, and at the laſt column to ſet down the whole number.

3	4	5
1 6 7 5 2	6 0 3 7	3 4 7 1 2 6
5 4 0 3 8	2 4 8 0	5 7 0 3 2 8
2 6 3 7 1	2 6 5 1	4 2 1 6 8 3

6	7	8
5 7 9 1	4 2 6 1 7	9 8 7 0
6 8 5 3	1 2 3 5 8	2 0 4 6
4 2 0 9	6 4 5	3 7 6 5
8 6 7 3	7 3	4 3 2 1
		7 8 1

9

7	3	5	2
3	0	9	8
5	6	7	5
1	2	9	8

10

8	6	8	2
3	5	7	1
9	4	2	6
3	6	7	8

11

2	6	8	7	1
6	5	7	3	4
8	3	4	7	5
3	2	7	8	6

12

6	3	9	8	7	5
4	6	8	2	3	7
2	8	7	5	4	1
6	7	8	5	4	0

13

3	4	5	6	7	8	9
9	8	6	4	3	5	1
7	3	8	7	9	5	2
9	6	7	4	9	8	0

14

6	3	5	7	0
		2	6	8
	4	5	3	
			9	8
		3	6	7
			6	0

15

3	5	0	0	0	0	0
		6	7	2	1	3
			7	8	6	7
					9	8
						7

Supplement to Addition.

THE attentive Scholar who has underftood, and ftill carries in his mind, what has already been taught him of Addition, will be able to anfwer his inftructor to the following.

QUESTIONS.

1. What is fimple addition ?
2. How do you place numbers to be added ?
3. Where do you begin the addition ?
4. How is the fum or amount of each column to be fet down ?
5. What do you obferve in regard to fetting down the fum of the laft column ?
6. Why do you carry for ten rather than any other number ?
7. How is addition proved ?
8. Of what ufe is addition ?

NOTE 1. Should the Learner find any difficulty in giving an anfwer to the above queftions, he is advifed to turn back and confult his Rule, with its Illuftrations.

NOTE. 2. In treating the Rules of Arithmetic the Scholar, in all inftances, is not particularly inftructed in the ufe and application of them to the purpofes of life. This is a point, however, to which his thoughts fhould be called ; therefore it is made a queftion here. A confideration of the Rule and of the queftions, which it involves, naturally fuggefts an anfwer. To confideration, therefore, let the Scholar apply himfelf. The mind acquires ftrength by exercife ; inftruction ought ever to be plain, but never fo full as to preclude a neceffity that the Scholar fhould in fome degree exercife his own thoughts ; it fhould be given in fuch a manner as to *force* him into fome reflections of his own.

EXERCISES.

1. What is the amount of 2801 dollars ; 765 dollars ; and of 397 dollars, when added together ? *Ans.* 3963 *dollars.*

2. Suppofe you lend a neighbor £210 at one time, £76 at another, £17 at another, and £9 at another, what is the fum lent ? *Ans.* £312.

NOTE. The Scholar who looks at greatnefs in his clafs will not be difcouraged by a little difficulty which may at firft occur in ftating his queftion, but will apply himfelf the more clofely to his Rule, and to thinking that if poffible, he may be able of himfelf to anfwer what another may be obliged to have taught him by his Inftructor.

3. WASHINGTON was born in the year of our Lord 1732; he was 67 years old when he died; in what year of our Lord did he die?

4. There are two numbers; the less number is 8761, the difference between the numbers is 597; what is the greatest number?

5. From the creation to the departure of the Israelites from Egypt was 2513 years; to the siege of Troy, 307 years more; to the building of Solomon's Temple, 180 years; to the building of Rome, 251 years; to the expulsion of the kings from Rome, 244 years; to the destruction of Carthage, 363 years; to the death of Julius Cæsar, 102 years; to the Christian æra, 44 years; required the time from the Creation to the Christian æra?

Ans. 4004 *years.*

6. At the late Census, taken A. D. 1800, the number of Inhabitants in the *New-England States* was as follows, *viz.* *Newhampshire* 183858; *Massachusetts*, 422845; *Maine*, 151719; *Rhode-Island*, 69122; *Connecticut*, 251002; *Vermont*, 154465; what was the number of Inhabitants at that time in *New-England?*

Ans. 1233011 *Inhabitants.*

§ 2. Simple Subtraction.

SIMPLE SUBTRACTION is the taking a less number from a greater of the same denomination, so as to shew the difference or remainder; as 5 taken from 8, there remains 3.

The greater number (8) is called the *Minuend*, the less number (5) the *Subtrahend*, and the difference (3) or what is left after subtraction, the *Remainder*.

RULE.

"Place the less number under the greater, units under units, tens under "tens, and so on. Draw a line below; then begin at the right hand, and "subtract each figure of the less number from the figure above it; and "place the remainder directly below. When the figure in the lower line "exceeds the figure above it, suppose 10 to be added to the upper figure; "but in this case you must add 1 to the under figure in the next column "before you subtract it. This is called *borrowing ten*."

PROOF.

Add the remainder and subtrahend together, and if the sum of them correspond with the minuend, the work is supposed to be right.

EXAMPLES.

Minuend	8 6 5 3
Subtrahend	5 2 7 1
Remainder	3 3 8 2
Proof	8 6 5 3

The numbers being placed with the larger uppermost, as the rule directs, I begin with the unit or right hand figure in the subtrahend, and say, 1 from 3 and there remain 2, which I set down, and proceeding to tens, or the next figure, I say 7 from 5 I cannot, I therefore borrow, or suppose 10 to be added to the upper figure (5) which make 15, then I say, 7 from 15, and there remain 8, which I set down; then proceeding to the next place, I say, 1 which I borrowed to 2 is 3, and 3 from 6 and there remain 3; this I set down, and in the next place I say 5 from 8 and there remain 3, which I set down, and the work is done.

PROOF. I add the remainder to the subtrahend; on finding the sum just equal to the minuend, and suppose the work to be right.

Note. The reason of *borrowing ten* will appear if we consider, that, when two numbers are equally increased by adding the same to both, their difference will be equal. Thus, the difference between 3 and 5 is 2; add the number 10 to each of these figures (3 and 5) they become 13 and 15, still the difference is 2. When we proceed as above directed, we add or suppose to be added, 10 to the *minuend*, and we likewise add 1 to the next higher place of the *subtrahend*, which is just equal in value to 10 of the lower place.

2. From	3 2 7 8 6 5 3 2 1 4 6 5	the minuend,
Take	1 0 6 7 9 3 6 1 2 3 4 2	the subtrahend.
		Remainder.

3. From 3 1 6 dollars,
Take 1 0 7 dollars.

Remainder

Proof

4. From 7 0 6 3 5 guineas,
Take 2 7 8 3 guineas.

Remainder

Proof

5. From 1 0 2 3 6 7 4 2 3 1 7 9 8 1 0
Take 8 7 9 1 2 8 4 5 0 6 7 0 3 2

Remainder

6. From 3 7 5 1 dollars, take 1 6 7 4 dollars.
Write the less number under the greater, with units under units, &c. as the rule directs.

Thus, 3 7 5 1
1 6 7 4

Remainder

7. From 2673105, the minuend,
Take 178932, the subtrahend.

OPERATION.

Minuend

Subtrahend

8. From 10000000

Subtract 9999999

The distance of time since any remarkable event, may be found by subtracting the date thereof from the present year.

EX.

How long since the American Independence, which was declared in 1776?

1 8 1 0 present time.
1 7 7 6 date of the Ind.

Ans. 3 4 *years since.*

So, likewise, the distance of time from the occurrence of one thing to that of another, may be found by subtracting the date of the thing first happening, from that of the last. EX.

How long from the discovery of America, by Columbus, 1492, to the commencement of the war, 1775, which gained our Independence?

1 7 7 5
1 4 9 2

Ans. 2 8 3 years.

Supplement to Subtraction.

QUESTIONS.

1. What is Simple Subtraction?
2. How many numbers must there be given to perform that operation?
3. How must the given numbers be placed?
4. What are they called?
5. When the figure in the lower number is greater than that of the upper number, from which it is to be taken, what is to be done?
6. How does it appear, that in subtracting a less number from a greater, the occasional *borrowing of ten*, does not affect the difference between these two numbers?
7. How is Subtraction proved?
8. When, and how may Subtraction be of use to a man engaged in the pursuits of life?

EXERCISES.

1. What is the difference between 78360 and 5841? *Ans.* 72939.

2. From a piece of cloth that measured 691 yards, there were sold 278 yards; how many yards should there remain? *Ans.* 413.

Note. In case of *borrowing ten*, it is a matter of indifference, as it respects the operation, whether we suppose 10 to be added to the upper figure, and from the sum subtract the lower figure and set down the difference; or as Mr. Pike directs, first, subtract the lower figure from 10, and adding the difference to the figure above, set down the sum of this difference and the upper figure. The latter method may, perhaps, be thought more easy, but it is conceived, that it does not lead the understanding of youth so directly into the nature of the operation as the former.

3. There are two numbers whose difference is 3 7 5, the greater number is 8 6 2; I demand the less? *Ans.* 487.

4. What number is that, which taken from 1 7 5 leaves 96? *Ans.* 79.

5. The capture of Gen. BURGOYNE and his army happened in the year 1 7 7 7, that of Cornwallis in 1 7 8 1; how many years between these events? *Ans.* 4 years.

6. Suppose you should lend a neighbor 2 7 6 5 dollars at a certain time, and he should pay you 973 at another; how much would remain due? *Ans.* 1 7 9 2 dollars.

7. Supposing a man to have been born in the year 1 7 4 5, how old was he in 1 7 9 9? *Ans.* 5 4 years.

8. What number is that to which if you add 7 8 9 it will become 6 3 5 0? *Ans.* 5 5 6 1.

9. Suppose a man to have been 63 years old in the year 1 8 0 1; in what year was he born? *Ans.* In the year 1 7 3 8.

10. King Charles, the martyr, was beheaded, 1 6 8 4, how many years is it since?

§ 3. Simple Multiplication.

SIMPLE MULTIPLICATION teaches, having two numbers given of the ſame denomination, to find a third which ſhall contain either of the two given numbers as many times as the other contains a unit.—Thus, 8 multiplied by 5, or 5 times 8 is 40.—The given numbers (8 and 5) ſpoken of together are called *Factors*. Spoken of ſeparately, the firſt or largeſt number, (8) or number to be multiplied, is called the *Multiplicand*; the leſs number, (5) or number to multiply by, is called the *Multiplier*, and the amount, (40) the *Product*.

This operation is nothing elſe than the addition of the ſame number ſeveral times repeated. If we mark 8 five times underneath each other and add them, the ſum is 40, equal to the product of 5 and 8 multiplied together. But as this kind of addition is of frequent and extenſive uſe, in order to ſhorten the operation, we mark down the number only once, and conceive it to be repeated as often as there are units in the multiplier.

8
8
8
8
8
—
40

Before any progreſs can be made in this rule, the following Table muſt be committed perfectly to memory.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

By this Table the product of any two figures will be found in that ſquare which is on a line with the one and directly under the other. Thus, 56, the product of 7 and 8, will be found on a line with 7 and under 8: ſo 2 times 2 is 4; 3 times 3 is 9, &c.—In this way the table muſt be learned and remembered.

RULE.

1. Place the numbers as in Subtraction, the larger number uppermost with units under units, &c. then draw a line below.

2. *When the multiplier does not exceed* 12, begin at the right hand of the multiplicand, and multiply each figure contained in it by the multiplier, setting down all over *even tens*, and carrying as in addition.

3. *When the multiplier exceeds* 12, multiply by each figure separately, first by the *units* of the multiplier, as directed above, then by the *tens*, and the other figures in their order, remembering always, to place the first figure of each product directly under the figure by which you multiply; having gone through in this manner with each figure in the multiplier, add their several products together, and the sum of them will be the product required.

EXAMPLES.

1. Multiply 5 2 9 1 by 3

Operation.

5 2 9 1	Multiplicand.
3	Multiplier.
1 5 8 7 3	Product.

The numbers being placed as seen under the operation, say—3 times 1 is 3; which set down directly under the multiplier; then 3 times 9 is 27; set down 7 and carry 2. Again, 3 times 2 is 6, and 2 I carry is 8; set down 8; then lastly, 3 times 5 is 15, which set down, and the work is done.

2. Multiply 3 6 0 2 by 12

Operation.

3 6 0 2
1 2
4 3 2 2 4

The numbers being properly placed, proceed thus, 12 times 2 is 24, set down 4 and carry 2; 12 times 0 is nothing, but 2 I carried is 2, which set down; then 12 times 6 is 72, set down 2 and carry 7; lastly, 12 times 3 is 36, and 7 I carried is 43, set down the whole number.

5. What is the product of 4175 multiplied by 37?

Place the factors thus, { 4 1 7 5 Multiplicand. / 3 7 Multiplier. }

4 1 7 5	Multiplicand.
3 7	Multiplier.
2 9 2 2 5	Product by the *units* (7) of the multiplier.
1 2 5 2 5	Product by the *tens* (3)
1 5 4 4 7 5	Product or answer.

In this example, as the Multiplier exceeds 12, therefore, you must multiply by each figure separately. First, by the units (7) just in the manner of the other examples. Secondly, by the tens (3) in the same way excepting only, that the first figure of the product in the multiplication by 3, must be placed under the 3, that is, under the figure by which you multiply. Lastly, add these two products together, the sum of them is the answer.

PROOF.

Multiplication may be proved by Division, but a method more concise

and easy, often practised by accountants, and which I shall recommend, is called

Casting out the 9's.

Casting out the 9's from any sum or number, is the exhausting of that number by the figure 9, till there is nothing left of it but a remainder, or *excess* over *even nines,* which remainder or excess is the thing sought.

How to cast out the 9's.

Whatever method may be adopted, this in effect, is nothing else than dividing the number by 9. The operation, however, would be tedious, as naturally practised by division; besides, as yet, we do not suppose the learner acquainted with it. A shorter and more successful way is the following

METHOD.

Beginning at the right hand of the number, add the figures, and when the sum exceeds 9, drop the sum and begin anew by adding, first, the figures, which would express it. Pass by the nines, and when the sum comes out exactly 9, neglect it; what remains after the last addition will be the remainder sought. †

EXAMPLES.

If it be required to cast the 9's out of 576394, proceed thus;—5 to 7 is 12; which sum *(twelve)* as it exceeds 9 you must drop, and begin anew, first add the figures (12) which would express *twelve,* saying 1 to 2 is 3, and (proceeding with the other figures, which remain to be added) 6 is 9, being

---

† This method of *casting out the 9's* succeds on a

PRINCIPLE,

That every § figure, in rising from the place of units to that of tens, takes to itself the addition of 9 times its value. The same from tens to hundreds, &c.

Consequently, if any figure, for instance *4, be removed from units place and divided by 9, it will leave a remainder of 4; the same of any other ‡ figure, removed and divided by 9, it will leave a remainder of ITSELF, and that only.

Therefore, if any †† number be divided by 9; or, the figures which express that number be added together, and the sum of them divided by 9, the remainder will be equal.

§ *Made evident thus;*—1 *in the place of units is the expression of an individual or one, in the place of tens,* (10) *it is the expression of ten individuals or ones; therefore taking* 1 *(one) its signification in units place, from* 10 *(ten) its signification in tens place, leaves* 9, *the increase of* 1, *or* 9 *times its value, in rising from the place of units to that of tens.*

* 4 *removed from units place by a cypher is* 40, *which divided by* 9 *leaves* 4 *(*4 *times* 9 *is* 36.*)*

‡ 6 *removed by a cypher is* 60, *which divided by* 9 *leaves a remainder of* 6; *or* 600 *divided by* 9, *still the remainder is* 6, *the remainder always begins the same figure whatever may be the place of its removal if divided by* 9.

†† *Thus,* 5683 *divided by* 9, *the remainder is* 4; *let the figures which express the number* 5683 *be added together*—5 *to* 6 *is* 11 *and* 8 *is* 19 *and* 3 *is* 22, *which number* (22) *divided by* 9 *leaves a remainder of* 4, *the same as when the number* 5683 *was divided by* 9.

These properties of the figure 9 belong to none other of the Digits, excepting to the figure 3, and this figure (3) possesses them in consequence only of being an even part of 9.

exactly nine, neglect it, and begin again; 3 to 9 is *twelve*; again, drop the sum *(twelve)* and add the figures (12) which would express it, 1 to 2 is 3 and 4 is 7, which sum (7) is the remainder after the last addition, or the thing sought, and is the remainder that would be left after dividing the sum 576394 by 9.

To prove Multiplication.

Cast the 9's out of the *Multiplicand* by the foregoing method, and mark down the remainder; cast the 9's out of the *Multiplier*, mark the remainder, then multiply the remainder first obtained by this last remainder, and cast the 9's out of the *product*; also, cast the 9's out of the *answer* or product of the Multiplicand and Multiplier, then if these two last remainders correspond, the work is supposed to be right.

EXAMPLES.

Let 7 6 5 3 0 2 be multiplied by 65.

Cast out the 9's from	7 6 5 3 0 2	Remainder 5	Remainders multiplied together.
——— from	6 5	Remainder 2	
	3 8 2 6 5 1 0	9's from 10 Rem. 1	Correspon-ding with each other.
	4 5 9 1 8 1 2		
9's out of	4 9 7 4 4 6 3 0	Remainder 1	

There is nothing more easy than proving multiplication by this method so soon as the scholar shall have given it such attention, as to make it a little familiar.

Note. Should the Multiplier or Multiplicand, either or both, be less than 9, they are to be taken as the remainders.

The examples which follow are to be wrought and proved according to the illustrations already given.

4. Multiply 6 2 3 7 5
By 8 4 Proof.

5 2 3 9 5 0 0 Product.

5. Mult. 3 7 8 4 6
By 2 3 5 } Product, 8893910

6. What is the product of 14356 multiplied by 648 ? *Ans.* 9302688

7. What is the product of 93956 multiplied by 8704 ? *Ans.* 817793024

8. Multiply	3	4	6	2	3	2	1	}	Product, 334058579364
By			9	6	4	8	4	}	

Contractions and Varieties in Multiplication.

Any number which may be produced by the multiplication of two or more numbers, is called a *composite number.* Thus 15 which arises from the multiplication of 5 and 3 (3 times 5 is 15) is a composite number; and these numbers, 5 and 3, are called *component parts*. Therefore,

1. *If the Multiplier be a composite number*, multiply first by one of the component parts, and that product by the other; the last product will be the answer sought.

EXAMPLES.

1. Multiply 6 7 by 1 5.

OPERATION.

```
   6 7
     5 one of the component parts.
 -----
 3 3 5
     3 the other component part.
 -----
1 0 0 5 Product of 67 multiplied by 15.
```

2. Multiply 367 by 48, Product, 17616.

OPERATION.

Consider first, what two numbers multiplied together will produce 48; that is, what are the component parts of 48? Answer, 6 and 8 (6 times 8 is 48) therefore, multiply 367 first by one of the component parts, and the product thence arising by the other; the last product will be the answer sought.

3. Mult. 583 by 56. Prod. 32648.

OPERATION.

4. Mult. 1086 by 72. Prod. 78192.

OPERATION.

2. "*When there are cyphers on the right hand of either the Multiplicand or Multiplier, or both*, neglect those cyphers; then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both factors."

EXAMPLES.

1. Multiply 65430 by 5200

OPERATION.

```
  6 5 4 3 0
      5 2 0 0
  -----------



-----------------
3 4 0 2 3 6 0 0 0
```

Here in the multiplication of 65430 by 5200, the cyphers are seen neglected, and regard paid only to the significant figures. To the product are prefixed 3 cyphers; equal to the number of cyphers neglected in the factors.

```
2. Mult. 3 6 5 0 0
   By      7 3 0
         -------



     ---------------
Prod. 2 6 6 4 5 0 0 0
```

3. Mult. 78000 by 600.
Product, 46800000.

3. *When there are cyphers between the significant figures of the Multiplier,* omit the cyphers, and multiply by the significant figures only, placing the first figure of each product directly under the figure by which you multiply, and adding the products together, the sum of them will be the product of the given numbers.

EXAMPLES.

1. Mult. 154326 by 3007.

OPERATION.

```
     1 5 4 3 2 6
         3 0 0 7
     -----------
   1 0 8 0 2 8 2
 4 6 2 9 7 8
 ---------------
 4 6 4 0 5 8 2 8 2
```

In this example, the cyphers in the multiplier are neglected, and 154326 multiplied only by 7 and by 3, taking care to place the figure in each product directly under the figure from which it was obtained.

2.

```
       3 4 5 7
         3 0 2
       -------



 -------------
 1 0 4 4 0 1 4
```

3.

```
4 8 9 7 6 8 5 0
    4 0 0 0 3 0
---------------



-------------------------
1 9 5 9 2 2 0 9 3 0 5 5 0 0
```

4. *When the Multiplier is* 9, 99, *or any number of* 9's, annex as many cyphers to the Multiplicand, and from the number thus produced, subtract the multiplicand, the remainder will be the product.

EXAMPLES.

1. Mult. 6547 by 999.

OPERATION.

```
6 5 4 7 0 0 0
      6 5 4 7
-------------
6 5 4 0 4 5 3
```

Write down the Multiplicand, place as many cyphers at the right hand as there are 9's in the multiplier for a *minuend*, underneath write again the multiplicand for a *subtrahend*, subtract, and the remainder is the product of 6547 multiplied by 999.

2.

6 4 7 3 } Product, 640827
9 9 }

3.

7 0 2 1 } Prod. 695079
9 9 }

4

5 3 8 4 9 7 6 } Prod. 53844375024.
9 9 9 9 }

Supplement to Multiplication.

QUESTIONS.

1. What is Simple Multiplication?
2. How many numbers are required to perform that operation?
3. Collectively, or together, what are the given numbers called?
4. Separately, what are they called?
5. What is the result, or number sought, called?
6. In what order must the given numbers be placed for multiplication?
7. How do you proceed when the multiplier is less than 12?
8. When the multiplier exceeds 12, what is the method of procedure?
9. What is a composite number?
10. What is to be understood by the component parts of any number?
11. How do you proceed when the multiplier is a composite number?
12. When there are cyphers on the right hand of the multiplier, multiplicand, either or both, what is to be done?
13. When there are cyphers between the significant figures of the multiplier how are they to be treated?
14. When the multiplier consists of 9's, how may the operation be contracted?
15. How is Multiplication proved?
16. By what method do you proceed in casting out the 9's from any number?
17. How is Multiplication proved by casting out the 9's?
18. Of what use is Multiplication?

EXERCISES.

1. What sum of money must be divided between 27 men so that each may receive 115 dollars? *Ans.* 3105.

NOTE. The scholar's business in all questions for Arithmetical operations, is wholly with the numbers given; these are never less than *two;* they may be more; and these numbers, *in one way or another,* are always to be made use of to find the answer. To these therefore, he must direct his attention, and carefully consider what is proposed by the question, to be known.

2. An army of 10700 men having plundered a city, took ſo much money, that when it was ſhared among them, each man received 46 dollars; what was the ſum of money taken?

Ans. 492200

3. There were 175 men employed to finiſh a piece of work, for which each man was to receive 13 dollars; what did they all receive?

Ans. 2275

4. There is a certain town which contains 145 houſes, each houſe two families, and each family 6 inhabitants; how many are the Inhabitants of that town. *Ans.* 1740

5. If a man earn 2 dollars per week, how much will he earn in 5 years, there being 52 weeks in a year? *Ans.* 520 *dollars.*

6. How much wheat will 36 men thraſh in 37 days, at 5 buſhels per day, each man? *Ans.* 6660 *buſhels.*

7. If the price of wheat be 1 dollar per buſhel and 4 buſhels of wheat make 1 barrel of flour, what will be the price of 175 barrels of flour?

Ans. 700 *dollars.*

§ 4. Simple Diviſion.

SIMPLE DIVISION teaches, having two numbers given of the ſame denomination, to find how many times one of the given numbers contains the other. Thus, it may be required to know how many times 21 contain 7, the anſwer is 3 times. The larger number (21) or number to be divided, is called the *Dividend;* the leſs number, (7) or number to divide by, is called the *Diviſor;* and the anſwer obtained, (3) the *Quotient.*

After the operation, ſhould there be any thing left of the dividend, it is called the *Remainder.* This part, however, is uncertain; ſometimes there is no remainder. When it does happen, it will always be leſs than the diviſor, if the work be right, and the ſame name with the dividend.

RULE.

1. "Aſſume as many figures on the left hand of the dividend, as contain "the diviſor once or oftener; find how many times they contain it, and "place the anſwer as the higheſt figure of the quotient.

2. "Multiply the diviſor by the figure you have found, and place the "product under that part of the dividend from which it was obtained.

3. "Subtract the product from the figures above it.

4. "Bring down the next figure of the dividend to the remainder, and di-"vide the number it makes up as before."

When you have brought down a figure to the remainder, if the number it makes up be ſtill leſs than the diviſor, a cypher muſt be placed in the quotient, and another figure brought down.

EXAMPLES.

1. Divide 127 by 5.

```
Diviſor. Dividend. Quotient.
   5)     1 2 7   ( 2 5
          1 0
          ———
            2 7
            2 5
          ———
            2 Remainder.
```

The parts in Diviſion are to ſtand thus, the dividend in the middle, the diviſor on the left hand, the quotient on the right, with a half parentheſis ſeparating them from the dividend.

Proceed in this operation thus.—It being evident that the diviſor (5) cannot be contained in the firſt figure (1) of the dividend, therefore, aſſume the two firſt figures (12) and inquire how often 5 is contained in 12, finding it to be 2 times, place 2 in the quotient, and multiply the diviſor by it, ſaying 2 times 5 is 10, and place the ſum (10) directly under 12 in the dividend. Subtract 10 from 12, and to the remainder (2) bring down the next figure (7) at the right hand, making with the remainder (2) 27. Again inquire how many times 5 in 27; 5 times; place 5 in the quotient, multiply the diviſor, (5) by this laſt quotient figure (5) ſaying, 5 times 5 is 25, place the ſum (25) under 27, ſubtract, and the work is done. Hence it appears that 127 contains 5, 25 times, with a remainder of 2, which was left after the laſt ſubtraction.

This Rule, perhaps, at firſt will appear intricate to the young ſtudent, although it is attended with no difficulty. His liability to errors will chiefly ariſe from the diverſity of proceedings. To aſſiſt his recollection let him notice, that

The ſteps of Diviſion are four { 1. Find how many times, &c. 2. Multiply. 3. Subtract. 4. Bring down. }

E

It is sometimes practised to make a point (.) under the figures in the Dividend, as they are brought down, in order to prevent mistakes.

When the divisor is a large number, it cannot always certainly be known how many times it may be taken in the figures which are assumed on the left hand of the dividend till after the first steps in division are gone over, but the learner must try so many times as his judgment may best dictate, and after he has multiplied, if the product be greater than the number assumed, or that number in which the divisor is taken, then it may always be known that the quotient figure is too large, if after he has multiplied and subtracted, the remainder be greater than the divisor, then the quotient figure is not large enough, he must then suppose a greater number of times, and proceed again. This at first may occasion some perplexity, but the attentive learner after some practice, will generally hit on the right number.

2. Let it be required to divide 7012 by 52.

OPERATION.

```
Divisor. Dividend. Quotient.
 5 2) 7 0 1 2 (1 3 4
      5 2
      -----
      1 8 1
      1 5 6
      -----
        2 5 2
        2 0 8
        -----
          4 4 Remainder.
```

In this operation it is left for the scholar to trace the steps of procedure without having them particularly pointed out to him by words.

PROOF.

Division may be proved by multiplication.

RULE.

"Multiply the Divisor and Quotient together, and add the remainder, if "there be any, to the product; if the work be right, the sum will be equal "to the Dividend."

Take the last example.

```
The Quotient was 134 } Multiply them together.
The Divisor       52 }
                 ----
                 268
                670
                  44 Remainder added.
                ----
                7012 Equal to the Dividend.
```

Another and more expeditious way of proving division is

By casting out the 9's.

Cast out the 9's from the Divisor and the Quotient, multiply the results, and to the product add the remainder if any after division; from the sum of these cast out the 9's, also cast out the 9's from the Dividend, and if the two last results agree, the work is right.

3. Divide 17354 by 86.

OPERATION.

Divisor. Dividend. Quotient.

```
86)17354(201
   172..
   -----
     154
      86
     ---
      68 Rem.
```

PROOF.

9's out of *(Divis.)* 86. Rem. 5 } Multiplied together.
(Quot.) 201. Rem. 3 }

15

Remainder 68 added.

9's out of 83 Rem. 2 } Agreeing together.
9's out of *(Divid.)* 17354 Rem. 2 }

4. Divide 153598 by 29. *Quotient,* 5296. *Rem.* 14

OPERATION.

29)153598(

5. Divide 30114 by 63. *Quotient,* 478

6. Divide 974932 by 365. *Quotient*, 2671. *Remainder*, 17.

7. Divide 3228242 dollars equally among 563 men; how many dollars muſt each man receive? *Ans.* 5734

From a view of the queſtion it is evident, that the dollars muſt be divided into as many parts as there are men to receive them; conſequently the number of dollars muſt be made the *dividend*, and the number of men the diviſor; the quotient will then ſhew how many dollars each man muſt receive.

8. How many times does 1030603615 contain 3215? *Ans.* 320561 *times.*

Contractions and Varieties in Division.

1. *When the Divisor does not exceed* 12, the operation may be performed without setting down any figures excepting the quotient, by carrying the computation in the mind. The units which would remain after subtracting the product of the quotient figure and the divisor from the figures assumed of the dividend, must be accounted so many *tens*, and be supposed to stand at the left hand of the next figure in the dividend, then consider again how often the divisor may be had in the sum of them. Proceed in this way till all the figures in the dividend have been divided. This is called *Short Division.*

EXAMPLES.

1. Divide 732 by 3

OPERATION.

3) 7 3 2 (2 4 4

Here I say, how often 3 in 7; knowing it to be 2 times, I place 2 in the quotient, then considering that the quotient figure (2) and the divisor (3) multiplied together would be 6, and that this product (6) subtracted from 7, in the dividend, would leave 1, I then consider this remainder (1) as standing at the left hand of the next figure (3) of the dividend, which together make 13. I now say, how many times 3 in 13—4 times, therefore I place 4 in the quotient, which multiplied into the divisor (3) would be 12, and 12 subtracted from 13 would leave 1, which considered as standing at the left hand of the next or last figure (2) of the dividend, would make 12; again, how many times 3 in 12—4 times—I then place 4 in the quotient, which multiplied into the divisor (3) is 12; this product (12) I consider as subtracted from 12, I find there will be no remainder, and the work is done.

Note. The quotient may stand as it is seen in the example, or it may be placed under the dividend, thus,

$$\begin{array}{r} 3)\ 7\ 3\ 2 \\ \hline 2\ 4\ 4 \end{array}$$

2. Divide 37426 by 7.

OPERATION.

7) 3 7 4 2 6

Quotient, 5 3 4 6 *Rem*. 4.

Here I say, how often 7 in 37? 5 times and 2 remain; then how often 7 in 24? 3 times, and 3 remain; how often 7 in 32? 4 times and 4 remain, lastly, how often 7 in 46? 6 times and 4 remain.

3. Divide 12363 by 5. *Quot*. 2472. *Rem*. 3.

4. Divide 602571 by 8. *Quot*. 75321. *Rem*. 3.

2. *When there are cyphers at the right hand of the Divisor*, cut them off, also, cut off an equal number of figures from the right hand of the dividend, and place these figures at the right hand of the remainder.

EXAMPLES.

1. Divide 6203916 by 5700.

OPERATION.

```
57 | 00)62039 | 46(1088
       57...
       -----
        503
        456
        ---
         479
         456
         ---
        2346 Rem.
```

Here are two cyphers on the right hand of the divisor, which I cut off; also, I cut off two figures (46) from the dividend, and to the right hand of the remainder after the last division (23) I place the figures cut off from the dividend (46) which make the whole remainder 2346.

2. Divide 379432 by 6500. *Quot.* 58. *Rem.* 2432.

3. Divide 2764503721 by 83000. *Quot.* 33307. *Rem.* 22721.

3. *When the divisor is* 10, 100, 1000, *or* 1, *with any number of cyphers annexed,* cut off as many figures on the right hand of the dividend as there are cyphers in the divisor; the figures which remain of the dividend compose the *quotient*; those cut off, the *remainder*.

EXAMPLES.

1. Divide 1576 by 10.

OPERATION.

1 | 0) 1 5 7 | 6

Here we have one cypher in the divisor; therefore, cut off one figure (6) from the dividend; what remains, (157) is the *quotient*, and the figure cut off (6) the *remainder*.

2. Divide 3217 by 100.

OPERATION.

Quot. *Rem.*

1 | 0 0)3 2 | 1 7

Supplement to Division.

QUESTIONS.

1. What is Simple Divifion?
2. How many numbers muft there be given to perform that operation?
3. What are the given numbers called?
4. How are they to ftand for Divifion?
5. How many fteps are there in Divifion?
6. What is the firft? the fecond? the third? the fourth?
7. What is the refult or anfwer called?
8. Is there any other, or uncertain part pertaining to Divifion? What is it called?
9. Of what name or kind is the remainder?
10. What is fhort Divifion?
11. When there are cyphers at the right hand of the Divifor, what is to be done?
12. What do you do with figures cut off from the Dividend when there are cyphers cut off from the Divifor?
13. When the Divifor is 10, 100, or 1 with any number of cyphers annexed, how may the operation be contracted?
14. How many ways may Divifion be proved?
15. How is Divifion proved by Multiplication?
16. How may Divifion be proved by cafting out the 9's?
17. Of what ufe is Divifion?

EXERCISES.

1. Suppofe an eftate of 36582 dollars to be divided among 13 fons, how much would each one receive?
Ans. 2814 *dollars.*

2. An army of 15000 men having plundered a city, and took 2825000 dollars? what was each man's fhare?
Ans. 175 *dollars.*

3. A certain number of men were concerned in the payment of 18950 dollars, and each man paid 25 dollars, what was the number of men? *Ans.* 758.

4. If 7412 eggs be packed in 34 casks, how many in a cask? *Ans.* 218.

5. A farm of 375 acres is let for 1125 dollars, how much does it pay per acre? *Ans.* 3 *dollars.*

6. A field of 27 acres produces 675 bushels of wheat; how much is that per acre? *Ans.* 25 *bushels.*

7. Supposing a man's income to be 2555 dollars a year; how much is that per day, there being 365 days in a year? *Ans.* 7 *dollars.*

8. What number must I multiply by 13, that the product may be 871? *Ans.* 67.

§ 5. Compound Addition.

COMPOUND ADDITION is the adding of numbers, which consist of articles of different value, as pounds, shillings, pence, and farthings, called *different denominations* ; the operations are to be regulated by the value of the articles which must be learned from the Tables.

RULES FOR COMPOUND ADDITION.

1. Place the numbers so that those of the same denomination may stand directly under each other.

2. Add the first column or denomination together, and carry for that number which it takes for the same denomination to make 1 of the next higher. Proceed in this manner with all the columns, till you come to the last, which must be added as in Simple Addition.

1. OF MONEY.

TABLE.

4	Farthings *qr.*	make one	Penny, *marked*	*d.*
12	Pence	make one	Shilling,	*s.*
20	Shillings	make one	Pound,	£.

EXAMPLES.

1. What is the sum of £61 17*s.* 5*d.*——£13 3*s.* 8*d.*——and of £5 16*s.* 11*d.* when added together?

OPERATION.

£.	*s.*	*d.*	
61	17	5	Those numbers of the same denomination placed under each other, as the rule directs.
13	3	8	
5	16	11	
80	18	0	

I begin with the right hand column or that of pence, and having added it, find the sum of the numbers therein contained to be 24 ; now as 12 of this denomination make one of the next higher, or in other words, 12 pence make one shilling, therefore in this, or in the column of pence I must carry for 12 ; I now inquire how often 12 is contained in 24, the sum of the first column or that of pence ; knowing it to be 2 times and nothing over, I set down 0 under the column of pence, and carry 2 to that of shillings, to be added into the second column, saying 2 I carry to 6 is 8, and 3 is 11, and 7 is 18, and 10 to 18 is 28, and 10 again is 38 (for so each figure in tens place must be reckoned, 1 in that place being equal in value to 10 units.) Now as 20 shillings make one pound, therefore, in the column of shillings, I carry for 20 ; I then inquire how often 20 in 38 ? once, and 18 remains ; therefore, I set down directly under the column of shillings 18, what 38 contains more than 20, and for the even 20 carry 1 to pounds or the last column, which is to be added after the manner of Simple Addition.

Note. The method of proof for Compound Addition is the same as that of Simple Addition.

2

£.	s.	d.	qr.
18	4	11	1
26	15	3	0
8	1	7	3

3

£.	s.	d.	qr.
371	15	6	2
5	7	4	0
68	3	2	1
7	0	5	3

4. Supposing a man goes a journey, and on the first day

1802. May 14,	pays for a dinner - - - -	£0	1		6
	- - for oats for his horse -	0	0		6
	- - for sling - - - - -	0	1		2
15,	- - for supper and lodging -	0	2		0
	- - for horse keeping - -	0	1	1	0
	- - for bitters - - -	0	1		6
	- - for breakfast - - -	0	2		0
	- - to the barber for dressing .	0	1		6
	- - for dinner again and other refreshment	0	3		5

What were the gentleman's expenses?

5. Suppose I am indebted £. s. d.

To A. Thirty two pounds, fourteen shillings and ten pence.

— B. Forty one pounds, six shillings and eight pence.

— C. Seventy five pounds, eight shillings.

— D. Three pounds and nine pence.

How much is the debt? Ans.

6. A man purchases cattle; one yoke of oxen for £14 11 6; four cows for £18 19 7; and other stock to the amount of £21 5; what was the amount of the cattle purchased? Ans. £54 16 1

2. OF TROY WEIGHT.

By Troy Weight are weighed gold, ſilver, jewels, electuaries and liquors.

TABLE.

24 Grains *grs.*	make one	Penny weight, *marked*	pwt.
20 Penny weights	make one	Ounce,	oz.
12 Ounces	make one	Pound,	lb.

EXAMPLES.

1.

lb.	oz.	pwts.	grs.
70	10	13	4
3	9	7	16
28	0	0	5
7	3	6	2

Becauſe 24 grains make a penny weight, you carry one to the penny weight column for every 24 in the ſum of the column of grains; becauſe 20 penny weights make 1 ounce, you carry for 20 in pennyweights, and becauſe 12 ounces make one pound, you carry for 12 in the ounces. This is called carrying according to the value of the higher place.

2.

lb.	oz.	pwts.
161	7	19
6	5	6
28	0	14
	3	7

3.

lb.	oz.	pwts.	grs.
	7	14	23
	2	0	6
	11	13	5
	10	12	7

Note. The fineneſs of gold is tried by fire and is reckoned in *carats*, by which is underſtood the 24th part of any quantity; if it loſe nothing in the trial, it is ſaid to be 24 carats fine; if it loſe 2 carats, it is then 22 carats fine, which is the ſtandard for gold.

Silver which abides the fire without loſs is ſaid to be 12 ounces fine. The ſtandard for ſilver coin is 11 oz. 2 pwts. of fine ſilver, and 18 pwts. of copper melted together.

3. OF AVOIRDUPOIS WEIGHT.

By Avoirdupois weight are weighed all things of a coarſe and droſſy nature, as tea, ſugar, bread, flour, tallow, hay, leather, and all kind of metals except gold and ſilver.

TABLE.

16 Drams *dr.*	make one	Ounce, *marked*	oz.
16 Ounces		Pound, —	lb.
28 Pounds		Quarter of a hundred weight,	qr.
4 Quarters		100 weight, or 112 pound,	cwt.
20 Hund. weight		Ton, — —	T.

EXAMPLES.

1

T.	cwt.	qr.	lb.	oz.	dr.
186	3	2	25	11	8
4	17	0	23	7	6
9	8	3	7	2	5
2	3	1	16	5	11

2

T.	cwt.	qr.	lb.	oz.	dr.
801	3	2	25	11	8
7	19	3	14	5	6
86	2	0	6	0	15
3	7	1	0	6	4

NOTE. "175 Troy Ounces are preciſely equal to 192 Avoirdupois Ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb. Troy=5760 grains and 1 lb. Avoirdupois=7000 grains."

4. OF TIME.

TABLE.

60 Seconds *s.*	make one	Minute, *marked*	m.
60 Minutes		Hour,	h.
24 Hours		Day,	d.
7 Days		Week,	w.
4 Weeks		Month,	mo.
13 Months, 1 d. & 1 h.		*Julian Year,	Y.

EXAMPLES.

Y.	mo.	w.	d.	h.	m.	s.
16	10	3	6	23	57	43
28	7	2	5	16	28	32
39	6	1	3	17	38	11
87	4	0	1	14	15	17

2

Y.	mo.	w.	d.	h.	m.	s.
89	11	3	6	22	45	36
36	10	2	5	6	55	44
87	2	1	0	11	22	33
36	4	3	3	5	8	7

The number of days in each Calendar month may be remembered by the following verſe.

Thirty days hath September, April, June and November ;
February twenty eight alone ; all the reſt have thirty one.

* "The civil Solar year of 365 days being ſhort of the true by 5h. 48m. 57s. occaſioned the beginning of the year to run forward through the ſeaſon nearly one day in four years ; on this account Julius Cæſar ordained that one day ſhould be added to February every fourth year, by cauſing the 24th day to be reckoned twice ; and becauſe this 24th day was the ſixth (ſextillis) before the kalends of March, there were, in this year, two of theſe ſextiles, which gave the name of Biſſextile to this year, which being thus corrected, was, from thence, called the Julian year."

5. OF MOTION.

TABLE.

60 Seconds	make one	Prime Minute, *marked*	'' '
60 Minutes		Degree,	°
30 Degrees		Sign,	s.
12 Signs, or 360 degrees		The whole great circle of the Zodiac.	

EXAMPLES.

°	'	''
25	17	18
17	49	56
6	35	24
10	17	16

s	°	'	''
9	8	55	44
1	26	44	55
8	18	36	12
1	9	33	22

6. OF CLOTH MEASURE.

TABLE.

2 Inches, and one fourth	make one	Nail, *marked*	na.
4 Nails, or 9 inches		Quarter of a yard,	qr.
4 Quarters of a yard, or 36 inches		Yard,	yd.
3 Quarters of a yard, or 27 inches		Ell-Flemish,	E. Fl.
5 Quarters of a yard, or 45 inches		Ell-English,	E. E.
6 Quarters of a yard, or 54 inches		Ell-French,	E. Fr.
4 Quarters, 1 inch and one fifth, or 37 inches and one fifth		Ell-Scotch,	E. Sc.
3 Quarters and two thirds		Spanish Var.	

EXAMPLES.

1

Yds.	qr.	n.
614	3	3
36	1	2
7	0	1
1	2	0
15	3	2

2

E. E.	qr.	n.
19	3	2
56	1	3
7	2	2
63	0	1
18	2	0

7. OF LONG MEASURE.

By Long Measure are measured distances, or any thing where length is considered without regard to breadth.

TABLE.

3 Barley corns *bar.*		make one	Inch *marked*	in.
12 Inches			Foot,	ft.
3 Feet			Yard,	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet			Rod, Perch, or Pole,	pol.
40 Poles			Furlong,	fur.
8 Furlongs			Mile,	mile.
$69\frac{1}{2}$ Statute miles nearly			Degree of a great Circle.	
360 Degrees			A great Circle of the Earth.	deg.

EXAMPLES.

1.

Deg.	mi.	fur.	pol.	ft.	in.	bar.
168	57	7	26	15	11	2
124	53	6	18	7	6	1
79	36	1	7	9	10	0
4	7	3	0	3	2	1

2.

Deg.	mi.	fur.	pol.	ft.	in.
13	56	5	13	8	1
49	18	1	27	16	2
267	12	3	16	9	0
29	8	0	5	3	1

8. OF LAND OR SQUARE MEASURE.

By ſquare meaſure are meaſured all things that have length and breadth.

TABLE.

144 Inches	make one	Square foot,
9 Feet		—— Yard,
$30\frac{1}{4}$ yards, or $272\frac{1}{4}$ Feet		—— Pole,
40 Poles		—— Rood,
4 Roods, or 160 Rods, or 4840 yards,		—— Acre,
640 Acres		—— Mile.

EXAMPLES.

Acres.	rood.	pol.	ft.	in.
376	3	36	93	121
568	1	27	58	76
247	2	35	61	24

9. OF SOLID MEASURE.

By Solid Meaſure are meaſured all things that have length, breadth and thickneſs.

TABLE.

1728 Inches	make one	Foot,
27 Feet		Yard,
40 Feet of round timber, or 50 feet of hewn timber		Ton or Load,
128 ſolid feet, i. e. 8 in length 4 in breadth, and 4 in height		Cord of Wood.

EXAMPLES.

Ton.	ft.	in.	cord.	ft.	in.
65	37	229	39	118	1021
19	26	1207	3	56	437
36	17	54	18	72	659
57	38	6	29	86	124

10. OF WINE MEASURE.

By Wine measure are measured Rum, Brandy, Perry, Cider, Mead, Vinegar and Oil.

TABLE.

2 Pints *pts.*	make one	Quart,	*marked*	qts.
4 Quarts		Gallon,		gal.
10 Gallons		Anchor of Brandy,		anc.
18 Gallons		Runlet,		run.
$31\frac{1}{2}$ Gallons		Half a hogshead		$\frac{1}{2}$hhd.
42 Gallons		Tierce,		tier.
63 Gallons		Hogshead,		hhd.
2 Hogsheads		Pipe or Butt,		P. or B.
2 Pipes		Tun,		T.

EXAMPLES.

1.

Hhd.	gal.	qts.	pts.
39	52	3	1
16	27	1	0
35	12	0	1
29	38	2	0

2.

T.	hhd.	gal.	qts.	pts.
36	2	58	3	1
35	1	36	1	0
17	0	29	2	1
23	2	12	1	0

N. B. A PINT wine measure, is $28\frac{7}{8}$ cubic inches.

11. OF ALE OR BEER MEASURE.

TABLE.

2 Pints	make one	Quart, *marked*	qts.
4 Quarts		Gallon,	gal.
8 Gallons		Firkin of Ale in London, A	fir.
8½ Gallons		Firkin of Ale or Beer	
9 Gallons		Firkin of Beer in London, B.	fir.
2 Firkins		Kilderkin,	Kill.
2 Kilderkins		Barrel,	Bar.
1½ Barrel, or 54 Gallons		Hogshead of Beer,	hhd.
2 Barrels		Puncheon,	Pun.
3 Barrels or 2 hogsheads		Butt.	Butt.

EXAMPLES.

1.

Hhd.	gal.	qts.
327	48	2
28	50	3
173	24	1
27	16	0

2.

B.fr.	gal.	qts.
23	6	2
45	2	3
98	7	1
36	8	0

N. B. A pint, Beer meaſure, is 35¼ cubic inches.

6. OF DRY MEASURE.

By Dry Meaſure are meaſured all dry goods, ſuch as Corn, Wheat, Seed, Fruit, Roots, Salts, Coal, &c.

TABLE.

2 Pints	make one	Quart, *marked*	qts.
2 Quarts		Pottle,	pot.
2 Pottles		Gallon,	gal.
2 Gallons		Peck,	pk.
4 Pecks		Buſhel,	bu.
2 Buſhels		Strike,	ſtr.
2 Strikes		Coom,	co.
2 Cooms		Quarter,	qr.
4 Quarters		Chaldron,	ch.
4½ Quarters		Chaldron in London,	
5 Quarters		Wey,	wey.
2 Weys		Laſt,	laſt.

EXAMPLES.

1.

Bus.	pk.	qts.	pts.
27	2	6	1
18	3	7	0
20	0	1	1
19	1	3	0

2.

Ch.	bus.	pk.	qts.
37	16	2	5
26	28	3	7
18	15	1	0
17	25	3	6

N. B. A gallon, Dry Meaſure, contains $268\frac{4}{5}$ cubic inches.

The following are denominations of things counted by the Table.

12 particular things make 1 Dozen,
12 Dozen —— 1 Groſs,
12 Groſs or 144 dozen great Groſs.

ALSO

20 Particular things make one Score.

Denominations of meaſure not included in the Tables.

6 Points make 1 Line,
12 Lines — Inch,
4 Inches — Hand,
3 Hands — Foot,
66 Feet, or 4 Poles, a Gunter's Chain.
3 Miles League.

A hand is uſed to meaſure Horſes.—A Fathom, to meaſure depths—A League, in reckoning diſtances at Sea.

N. B. A Quintal of Fiſh weighs 1 Cwt. Avoirdupois.

§ 6 Compound Subtraction.

COMPOUND SUBTRACTION teaches to find the difference between two ſums of diverſe denominations.

RULE FOR COMPOUND SUBTRACTION.

" Place thoſe numbers under each other, which are of the ſame donomina-
" tion, the leſs being below the greater; begin with the leaſt denomination,
" and if it exceed the figure over it, borrow as many units as make one of the
" next greater; ſubtract it therefrom; and to the difference add the upper
" figure, remembering, always to add *one* to the next ſuperior denomination,
" for that which you borrowed.

PROOF. In the ſame manner as Simple Subtraction.

I. OF MONEY.

1. Suppoſing a man to have lent £185 10s. 7d. and to have received at gain of his money, £93 15s. how much remains due?

OPERATION.

1.

	£.	s.	d.
Lent	185	10	7
Received	93	15	0
Due	91	15	7
Proof	185	10	7

2.

	£.	s.
From	310	
Take	85	15

3.

	£.	s.	d.
Lent	637	17	8
Received at Sundry times.	163	2	5
	78	4	
	19	15	11
	139	6	8
	261	1	4
Received in all			
Yet due			

The ſeveral payments muſt firſt be added together, and their ſum ſubtracted from the ſum lent.

4. A certain man fold a lot of land for £735 11 6; he received at one time £61 5; at another time, £195 13 11 how much is there yet due?
Ans. £478 12 7.

2. OF TROY WEIGHT.

1.

	lb.	oz.	pwt.	gr.
From	76	8	16	13
Take	3	9	17	6
Remains				
Proof				

2.

lb.	oz.	pwt.
7	3	5
2	8	9

3. OF AVOIRDUPOIS WEIGHT.

1.

lb.	oz.	dr.
9	15	5
5	6	7

2.

T.	cwt.	qr.	lb.	oz.	dr.
6	11	1	14	7	3
1	5	1	16	9	8

4. OF TIME.

1.

Y.	mo.	w.	d.	h.	m.	s.
39	6	3	6	20	44	55
16	9	1	2	18	59	57

5. OF MOTION.

1.

°	′	″
16	27	33
8	34	23

2.

s	°	′
6	8	51
3	9	57

6. OF CLOTH MEASURE.

1.

Yds.	qr.	n.
27	1	2
16	1	3

2.

E. E.	qr.	n.
26	2	1
17	3	2

7. OF LONG MEASURE.

1.

Deg.	mi.	fur.	p.	yds.	ft.	in.	bar.
56	13	5	26	2	1	8	1
17	15	2	27	1	2	9	2

8. OF LAND OR SQUARE MEASURE.

1.

A.	R.	pol.
17	1	17
16	1	16

2.

pol.	ft.	in.
18	16	11
10	201	130

9. OF SOLID MEASURE.

1.

Tons.	ft.	in.
45	29	186
19	34	1237

2.

Cords.	ft.	in.
68	23	810
6	127	1529

10. OF WINE MEASURE.

1.

Hhd.	gal.	qts.
66	31	2
17	33	3

2.

Hhd.	Tun.	gal.
75	1	16
24	1	43

11. OF ALE AND BEER MEASURE.

1.

Hhd.	gal.	qts.
89	19	2
37	25	3

2.

Butt.	hhd.	gal.
63	1	16
29	1	10

12. OF DRY MEASURE.

Bu.	pk.	qts.
61	1	2
5	1	4

Chal.	bu.	pk.
171	18	1
76	22	2

THE

SCHOLAR'S ARITHMETIC.

OBSERVATIONS.

THE Scholar has now ſurveyed the *ground work* of Arithmetic. It has before been intimated, that the only way in which numbers can be affected is by the operations of Addition, Subtraction, Multiplication and Diviſion. Theſe rules have now been taught him, and the exerciſes in a ſupplement to each ſuggeſt their uſe and application to the purpoſes and concerns of life. Further, the *thing needful*, and that which diſtinguiſhes the Arithmetician, is to know how to proceed by application of *theſe four rules* to the ſolution of any arithmetical queſtion. To afford the Scholar this knowledge is the object of all ſucceeding rules.

SECTION II.

Rules eſſentially neceſſary for every perſon to fit and qualify them for the tranſaction of buſineſs.

Theſe are nine: REDUCTION, FRACTIONS,* FEDERAL MONEY, INTEREST, COMPOUND MULTIPLICATION, COMPOUND DIVISION, SINGLE RULE OF THREE, DOUBLE RULE OF THREE and PRACTICE.

A thorough knowledge of theſe rules is ſufficient for every ordinary occurrence in life. Short of this a perſon in any kind of buſineſs, will be liable to repeated embarraſſments. It is the extreme uſefulneſs of theſe rules which commends them to the attention of every Scholar.

* *Fractions are taken up here no further than is neceſſary to ſhew their ſignification, and to illuſtrate the principles of* FEDERAL MONEY.

§ 1. Reduction.

—xxx:✽:xxx—

" REDUCTION teaches to bring or exchange numbers of one denomi-
" nation to others of different denominations, retaining the ſame value."

It is of two kinds.

1. *When high denominations are to be brought into lower*, as pounds into ſhillings, pence, and farthings; it is then called REDUCTION DESCENDING, and is performed by *Multiplication.*

2. *When lower denominations are to be brought into higher*, as farthings into pence, or into pence, ſhillings and pounds; it is then called REDUCTION ASCENDING, and is performed by *Diviſion.*

REDUCTION DESCENDING.

RULE.

Multiply the higheſt denomination by that number which it takes of the next leſs to make one of that greater; ſo continue to do, till you have brought it as low as your queſtion requires.

PROOF. " Change the order of the queſtion, and divide your laſt product by the laſt multiplier, and ſo on."

EXAMPLES.

1. In £17 13s. 6d. 3qrs. how many farthings?

OPERATION.

```
    £.  s.  d.  qrs.
    17  13  6   3
    20 Shillings in a pound.
   ----
   353 Shillings in £17 13s.
    12 Pence in a ſhilling.
  -----
  4242 Pence in £17 13s. 6d.
     4 Farthings in a penny.
 ------
A. 16971 Farthings.
```

In this example, the higheſt denomination is pounds, the next leſs, is ſhillings, and becauſe 20 ſhillings make *one pound* therefore I multiply £17 by 20, increaſing the product by the addition of the given ſhillings (13) which it muſt be remembered, muſt always be done in like caſes; then, becauſe 12 pence make *one ſhilling*, I multiply the ſhillings, (353) by 12, adding in the given pence (6d.) laſtly becauſe 4 farthings make one penny, I multiply the pence (4242) by 4 and add in the given farthings (3qrs.) I then find, that in £17 13s. 6d. 3qrs. there are 16971 farthings.

PROOF.

```
 4) 16971
   ------
12) 4242  3qrs.
   ------
2|0) 35|3  6d.
   ------
   £17    13s.
```

To prove the above queſtion change the order of it, and it will ſtand thus; in 16971 farthings how many pounds?

Divide the laſt product by the laſt multiplier, the remainder will be farthings. Proceed in this way till all the ſteps of the operation have been retraced back; the laſt quotient with the remainders will be proof of the accuracy of the operation if they agree with the ſum given in the queſtion.

2. In £7 14s. 6d. 1qr. how many farthings? *An.* 7417 qrs.

3. In £7 6s. 4d. how many pence? *Ans.* 1756d.

4. In 29 Guineas, at 28s. how many farthings? *Ans.* 38976 qrs.

5. In £173 15s. how many sixpences? *Ans.* 6950.

6. In 12 crowns, at 6s. 7d. how many pence and farthings? *Ans.* 948d. 3792 qrs.

7. In 671 eagles, at 10 dollars each, how many shillings, three-pences, pence and farthings? *Ans.* 40260 *shill.* 161040 *three-pences*, 483120 *pence*, and 1932480 qrs.

REDUCTION ASCENDING.

RULE.

Divide the loweſt denomination given by that number which it takes of the ſame to *make one* of the next higher, and ſo continue to do, till you have brought it into that denomination which your queſtion requires.

EXAMPLES.

1. In 16971 farthings how many pounds?

OPERATION.

Farthings in a penny	4)16971	
Pence in a ſhilling	12)4242	3qrs.
Shillings in a pound	2|0)35|3	6d.
	£17	13s.

Ans. £17 13s. 6d. 3qrs.

Reduction deſcending and aſcending reciprocally prove each other.

2. In 1765 pence, how many pounds? *Ans.* £7 7s. 1d.

3. In 38976 farthings, how many guineas? *Ans.* 29.

4. In 6950 ſix-pences, how many pounds? *Ans.* £173 15s.

5. In 3792 farthings, how many crowns? *Ans.* 12.

6. In 48960 farthings, how many pence, three-pences, six-pences, and dollars ?
Ans. 12240 *pence*, 4080 *three-pences*, 2040 *six-pences*, 170 *dollars*.

7. In 6952 three-pences, how many pistoles at 22s. each ?
Ans. 79.

REDUCTION ASCENDING AND DESCENDING.

1. MONEY.

1. In 57 moidores, at 36s. each, how many dollars ?
Ans. 342 dollars.

In this question, the first step will be to bring the moidores into shillings ; lastly bring the shillings into dollars.

2. In 75 pistoles how many pounds ?
Ans. £82 10s.

3. In £73 how many guineas ?
Ans. 52 *guineas*, 4*s*.

4. In £63 and 5 guineas how many dollars ?
Ans. 233 dollars, 2s.

" *When it is required to know how many ſorts of coin of different values, and of equal number, are contained in any number of another kind*; reduce the ſeveral ſorts of coin into the loweſt denomination mentioned, and add them together for a diviſor; then reduce the money given, into the ſame denomination for a dividend, and the quotient ariſing from the diviſion will be the number required."

" NOTE. Obſerve the ſame direction in weights and meaſures."

1. In 54 guineas, how many pounds, dollars and ſhillings of each an equal number?

OPERATION.

```
      £1 is 20 ſhillings            54 guineas
A dollar is  6 ſhillings            28 Shillings is a guinea
             1 ſhilling            ---
            ---                    432
   Diviſor, 27 Shillings          108
                                  ---
                        Dividend, 1512 ſhillings.
```

```
27)1512(56 of each; that is, 54 guineas include the value of one pound,
   135       one dollar and one ſhilling, 56 times.
   ---
    162
    162
    ---
    000
```

2. In 172 moidores, how many eagles, dollars and nine-pences, of each the like number? *Ans.* 92 *of each, and* 68 *nine-pences over.*

2. TROY WEIGHT.

1. In 4lb. 5oz. 16pwts. how many grains?

OPERATION.

```
          lb.     oz.     pwts.
           4       5       16
          12 oz. in a pound.
         ----
          53 Ounces.
          20 pwts. in an ounce.
         ----
        1076 Penny weights.
          24 grs. in cne pwt.
        ----
        4304
       2152
       -----
Proof. 24)25824 Grains, the anfwer.
       20)1076  16 pwts.
          ----
       12)53    5 oz.
          ---
           4 lb.
```

2. In 10lb. of filver, how many fpoons, each weighing 5 oz. 10 pwts.?
Ans. 21 *fpoons, and* 90 *pwts. over.*

3. In 45681 grains of ſilver, how many pounds ?

OPERATION.

	20	12		
24)45681(	1903(	95(7lb.	Anſwer, 7lb.	11oz. 3pwts. 9grs.
24	180	84	12	
216	103	11 oz.	95	
216	100		20	
081	003 pwts.		1903	
72			24	
09 grs.			45681 Proof.	

4. In 4560 grains of ſilver, how many tea-ſpoons, each one ounce ?

Ans. $9\frac{1}{2}$ *tea-ſpoons.*

3. AVOIRDUPOIS WEIGHT.

	cwt.	qr.	lb.	oz	
1.	in 67	1	13	11	how many drams ?

4	PROOF.	
269	16)1931696	
28	16)120731	11 oz.
2165	28)7545	13 lb.
538	4)269	1 qr.
7545	67	Cwt.
16		
45281		
7545		
120731		
16		
724386		
120731		
1931696		

2. In 14048 oz. how many hundred weight? *Ans.* 7 C. 3qrs. 10lb.

3. In 470 boxes of Sugar, each 26lb. how many Cwt.?
Ans. 109 C. 0qrs. 12lb.

4. In 17Cwt. 1qr. 6lb. of Sugar, how many parcels, each 17lb.?
Ans. 114 parcels.

4. TIME.

1. In 121812 ſeconds, how many hours ?

OPERATION.	PROOF.		
6,0)121812	H.	m.	s.
	33	50	12
6,0)203|0 12 ſec.	60		
	2030		
Ans. 33h. 50 m. 12s.	60		
	121812		

2. Suppoſing a man to be 21 years old, how many ſeconds has he lived allowing 365 days, 6 hours to a year ? *Ans.* 662709600 ſeconds.

3. How many minutes from the commencement of the war between America and England, April 19, 1775, to the ſettlement of a general peace which took place Jan. 20, 1783 ? *Ans.* 4079160 *minutes.*

4. In 413280 minutes how many weeks? *Ans.* 41.

5. LONG MEASURE.

1. Reduce 16 miles to barley-corns.

OPERATION.

16 Miles.
8
———
128 Furlongs.
40
———
5120 Rods.
$5\frac{1}{2}$*
———
25600
2560
———
28160 Yards.
3
———
84480 Feet.
12
———
1013760 Inches.
3
———
Anfwer, 3041280 Barley-corns.

PROOF.

3)3041280
———
12)1013760
———
3)84480
———
† 11)28160
———
2560
2
———
4|0)512|0
———
8)128
———
16 Miles.

† Divide by 11 for $5\frac{1}{2}$ and multiply the quotient by 2. The reafon is becaufe $5\frac{1}{2}$ reduced to *half yards* is 11.

* To multiply by *one half* ($\frac{1}{2}$) i is only to take half the Multiplicand.

2. In 47520 feet, how many leagues? *Ans.* 3 *leagues.*

3. How many times does the wheel, which is 18 feet 6 inches in circumference, turn round in the diſtance of 150 miles?

Ans. 42810 times, and 180 inches over.

4. How many barley-corns will reach round the Globe, it being 360 degrees?

Ans. 4755801600.

6. LAND OR SQUARE MEASURE.

6. In 13 acres, 2 roods, how many poles?

OPERATION.

```
Ac.  r.
13   2
 4
---
 54
  40
-----
```

Ans. 2160 *Poles.*

PROOF.

```
4|0)216|0
---------
   4)54
   ----
13 Ac. 2 roods.
```

2. In 2852 rods how many acres? *Ans.* 17 *A.* 3 *R.* 12 *P.*

7. SOLID MEASURE.

1. In 1296000 solid inches, how many tons of hewn timber?

OPERATION.

```
                 5|0
1728)1296000(75|0
     12096   ----
     -----   15 Tons, the Answer.
      8640
      8640
      ----
        00
```

PROOF.

```
     15
     50
    ---
    750
   1728
   ----
   6000
  1500
 5250
 750
-------
1296000 Inches.
```

2. In 5529600 ſolid inches, how many cords of wood? *Ans.* 25.

8. DRY MEASURE.

1. In 75 buſhels of corn, how many pints?

OPERATION.

```
  75
   4
----
 300
   8
----
2400
   2
----
```

Ans. 4800 *pints.*

PROOF.

```
2)4800
------
8)2400
------
 4)300
------
    75 buſhels.
```

2. In 9376 quarts, how many buſhels? *Ans.* 293

It would be needleſs to give examples of Reduction in all the weights and meaſures. The underſtanding, which the attentive ſcholar muſt already have acquired of this rule, by help of the tables, will ever be ſufficient for his purpoſe.

Supplement to Reduction.

QUESTIONS.

1. WHAT is Reduction ?
2. Of how many kinds is Reduction ? What are they called ? Wherein do these kinds differ one from the other ? Which of these fundamental rules are employed in their operations ?
3. How is Reduction Descending performed ?
4. How is Reduction Ascending performed ?
5. When it is required to know how many sorts of coin, weights or measures of different values, of each an equal number, are contained in any other number of another kind, what is the method of procedure ?

EXERCISES.

1. In 36 guineas how many crowns ? *Ans.* 153 *crowns, and* 9*d. over.*

2. How many ſteps of 2 feet 5 inches each, will it require a man to take, going from Leominſter to Boſton, it being 43 miles.
Ans. 93948 *ſteps ; The laſt ſtep will carry him into the town* 12 *inches.*

3. Let 70 dollars be diſtributed among three men in ſuch a manner that as often as the firſt has 5*s.* the ſecond ſhall have 7*s.* and the third 9*s.* What will each one receive ? *Ans. Firſt* 16 *dolls.* 4*s.* *Second* 23 *dolls.* 2*s.* *Third* 30 *dolls.*

4. If a vinter be desirous to draw off a Pipe of Canary into bottles containing pints, quarts and 2 quarts, of each an equal number, how many must he have? *Ans.* 144 *of each.*

5. There are three fields; one contains 7 acres, another 10 acres and the other 12 acres and 1 rood; how many shares of 76 perches each, are contained in the whole? *Ans.* 61 *shares and* 44 *perches over.*

6. There are 106lb. of ſilver, the property of 3 men; of which A receives 17lb. 10oz. 19pwts. 19grs. of what remains, B ſhares 1oz. 7grs. ſo often as C ſhares 13pwts. What are the ſhares of B and C?

Anſwer, B's ſhare 53lb. 8oz. 5pwts. 5grs. C's ſhare 34lb. 4oz. 15pwts.

§ 2. Fractions.

WHEN the thing or things signified by figures are *whole ones*, then the figures which signify them are called *Integers* or *whole numbers*. But when only some parts of a thing are signified by figures, as *two thirds* of any thing, *five sixths*, *seven tenths*, *&c.* then the figures which signify *these parts* of a thing being the expression of some quantity *less than one*, are called FRACTIONS.

FRACTIONS are of two kinds, *Vulgar* and *Decimal*; they are distinguished by the manner of representing them; they also differ in their modes of operation.

VULGAR FRACTIONS.

To understand Vulgar Fractions, the learner must suppose an integer (or the number 1) divided into a number of equal parts; then any number of these parts being taken, would make a fraction, which would be represented by two numbers placed one directly over the other, with a short line between them, thus $\frac{2}{3}$ *two thirds*, $\frac{3}{5}$ *three fifths*, $\frac{7}{8}$ *seven eights*, *&c.*

EACH of these figures have a different name and a different signification. The figure below the line is called the Denominator and shews into how many parts an integer, or one individual of any thing is divided—the figure above the line is called the numerator and shews how many of those parts are signified by the fraction.

FOR illustration, suppose a silver plate to be divided into *nine equal parts*. Now one or more of these parts make a fraction, which will be represented by the figure 9 for a denominator placed underneath a short line shewing the plate to be divided into *nine equal parts*; and supposing *two* of those parts to be taken for the fraction, then the figure 2 must be placed directly above the 9 and over the line ($\frac{2}{9}$) for a Numerator, shewing that two of those parts are signified by the fraction, or *two ninths* of the plate. Now let 5 parts of this plate, which is divided into 9 parts, be given to John, his fraction would be $\frac{5}{9}$ *five ninths*; let 3 other parts be given to Harry, his fraction would be $\frac{3}{9}$ *three ninths*; there would then be one part of the plate remaining still (5 and 3 are 8) and this fraction would be expressed thus $\frac{1}{9}$ *one ninth*.

IN this way all vulgar fractions are written; the Denominator, or number below the line shewing into how many parts any thing is divided, and the numerator, or number above the line, shewing how many of those parts are taken, or signified by the fraction.

To ascertain whether the Learner understands what has now been taught him of fractions, let us again suppose a dollar to be cut into 13 equal parts;—let 2 of those parts be given to A; 4 to B; and 7 to C.

REQUIRED of the Learner that he should write $\left\{\begin{array}{l}\text{A's fraction——.}\\ \text{B's fraction——.}\\ \text{C's fraction——.}\end{array}\right.$

IT is from Division only that fractions arise in Arithmetical operations; the remainder after division is a portion of the Dividend undivided; and is always the Numerator to a fraction of which the Divisor is the Denominator. The Quotient is so many integers.

THE Arithmetic of Vulgar Fractions is tedious and even intricate to beginners. Besides, they are not of necessary use. We shall not, therefore, enter into any further consideration of them here. This difficulty arises chiefly from the variety of denominators; for when numbers are divided into different kinds, or

parts they cannot be eaſily compared. This conſideration gave riſe to the invention of

DECIMAL FRACTIONS.

Decimal fractions are alſo expreſſions of parts of an integer ; or, are in value ſomething leſs than *one* of any thing, whatever it may be, which is ſignified by them.

In decimals, an integer, or the number *one*, as 1 foot, 1 dollar, 1 year, &c. is conceived to be divided into *ten* equal parts, *(in vulgar fractions an integer may be divided into any number of parts)* and each of theſe parts is ſubdivided into *ten* leſſer parts, and ſo on. In this way, the denominator to a decimal fraction in all caſes, will be either 10, 100, 1000, or unity (1) with a number of cyphers annexed ; and this number of cyphers will always be equal to the number of places in the numerator. Thus, $\frac{6}{10}$ $\frac{7}{100}$ $\frac{685}{1000}$ are *Decimal Fractions*, of which the cyphers in the denominator of each are equal to the number of places in its own numerator.

“ As the denominator of a decimal fraction is always 10, 100, 1000, &c. “ the denominators need not be expreſſed ; for the numerator only may be “ made to expreſs the true value; for this purpoſe it is only required to “ write the numerator with a point, (,) before it, called a *ſeparatrix*, at the “ left hand to diſtinguiſh it from a whole number ; thus, $\frac{6}{10}$ is written ,6 ; “ $\frac{27}{100}$,27 ; $\frac{685}{1000}$, ,685, &c.

When integers and decimals are expreſſed together in the ſame ſum, that ſum is called a mixed number : Thus, 25,63 is a mixed number ; 25, or all the figures to the left hand of the ſeparatrix being integers, and ,63 or all the figures to the right hand of the ſame point being decimals.

The firſt figure on the right hand of the decimal point ſignifies tenth parts, the next hundredth parts, the next thouſandth parts, and ſo on.

,7 ſignifies ſeven tenth parts.
,07——ſeven hundredth parts.
,27——two tenth parts and ſeven hundredth parts ; or twenty-ſeven hundredths.
,357——three tenth parts, five hundredth parts, and ſeven thouſandth parts ; or, 327 thouſandths.
5,7——five and ſeven tenth parts.
5,007——five and ſeven thouſandths.

The value of each figure from unity, and the decreaſe of decimals toward the right hand, may be ſeen in the following

TABLE.

C Millions	X Millions	Millions	C Thouſands	X Thouſands	Thouſands	Hundreds	Tens	Units	Tenth parts	Hundredth parts	Thouſandth parts	X Thouſandth parts	C Thouſandth parts	Millionth parts	X Millionth parts	C Millionth parts
9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9

Cyphers placed to the right hand of decimals do not alter their value ; placed at the left hand, they diminiſh their value in a tenfold proportion.

ADDITION OF DECIMALS.

RULE.

" 1. PLACE the numbers whether mixed or pure decimals, under each " other, according to the value of their places."

" 2. Find their ſum as in whole numbers, and point off ſo many places for " decimals as are equal to the greateſt number of decimal places in any of the " given numbers."

EXAMPLES.

1. What is the amount of 73,612 guineas, 436 guineas, 3,27 guineas, ,8632 of a guinea, and 100,19 guineas when added together?

OPERATION.

```
 73,612
436,
  3,27
   ,8632
100,19
--------
```

Ans. 613,9352 *guineas.*

The decimals are arranged from the ſeparatrix towards the right hand, and the whole numbers from the ſame point towards the left hand. The greateſt number of decimal places in any of the numbers is four, conſequently, four figures in the product muſt be pointed off for decimals.

2.

```
345,601
   ,3724
 63,1
572,313
  7,5462
--------
```

3. Required the ſum of 37,821+546,35+8,4+37,325?

Ans. 629,896.

4. What is the ſum of three hundred twenty nine and ſeven tenths; thirty ſeven and one hundred ſixty two thouſandths, and ſixteen hundredths when added together?

Ans. 367,022.

5. Add ſix hundred and five thouſandths, and four thouſand and three hundredths?

Sum 4600,035.

NOTE. When the numerator has not ſo many places as the denominator has cyphers, prefix ſo many cyphers at the left hand as will make up the defect; ſo $\frac{5}{1000}$ is written thus, ,005, &c.

SUBTRACTION OF DECIMALS.

RULE.

" Place the numbers according to their value ; then ſubtract as in whole numbers, and point off the decimals as in addition."

EXAMPLES.

1. From 716,325 take 81,6201.

OPERATION.

From 716,325
Take 81,6201

Rem. 634,7049

2. From 119,1384 take 95,91.
Rem. 23,2284.

3. What is the difference between 287 and 3,115 ?
Ans. 283,885

4. From 67 take ,92
Rem. 66,08

All the operations in Decimal Fractons are extremely eaſy ; the only liability to error will be in placing the numbers and pointing off the decimals ; and here care will always be ſecurity againſt miſtakes.

MULTIPLICATION OF DECIMALS.

RULE.

" Whether they are mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers."

" 2. Point off ſo many figures from the product as there are decimal places in both the factors ; and if there be not ſo many decimal places in the product, ſupply the defect by prefixing cyphers."

EXAMPLES.

1. Multiply ,0261 by ,0035

OPERATION.

,0261
,0035

1305
783

,00009135 *Product.*

In this example, the decimals in the two factors taken together are *eight ;* the product falls ſhort of this number by *four* figures, conſequently, four cyphers are prefixed to the left hand of the product.

2. Multiply 31,72 by 65,3
Product, 2071,316

OPERATION.

```
31,72
 65,3
```

3. Multiply 25,238 by 12,17
Product, 307,14646

4. Multiply ,62 by ,04
Product, ,0248

5. Multiply 17,6 by ,75
Product, 13,2

DIVISION OF DECIMALS.

RULE.

"1. The places of decimal parts in the divisor and quotient counted together must be always equal to those in the dividend, therefore divide as in whole numbers, and from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

"2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

"3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness."

EXAMPLES.

Divide 2,735 by 51,2

OPERATION.

```
51,2)2,735(,0534+
     2 560
     -----
      1750
      1536
      ----
       2140
       2048
       ----
         92
```

In this example, there are *five decimals* in the dividend (counting the two cyphers which were added to the remainder of the dividend after the first division) that the decimals in the divisor and quotient counted together may equal that number, a cypher is prefixed to the left hand of the quotient.

In division of decimals it is proper to add cyphers so long as there continues to be a remainder, this however is not practised, nor is it necessary ; four or five decimals being sufficiently accurate for most calculations.

2. Divide 3156,293 by 25,17.
Quotient, 1253+

The Scholar is requested to point the following example as the rule directs.

3. Divide 5737 by 13,3
Quotient, 431653+

4. Divide 173948 by ,375.
Quotient, 463861+

5. Divide 2 by 53,1
Quotient, 037+

REDUCTION OF DECIMALS.

CASE 1.

To reduce Vulgar Fractions to Decimals.

RULE.

ANNEX a cypher to the numerator and divide it by the denominator, annexing a cypher continually to the remainder. The quotient will be the decimal required.

EXAMPLES.

1. Reduce $\frac{3}{5}$ to a decimal.

OPERATION.

```
5)3,0(,6 Answer.
  30
  ---
  00
```

The numerator in these operations is considered as an integer, and always requires the decimal point to be placed immediately after it, the cyphers annexed occupy the places of decimals, the quotient must be pointed off according to the rule in Division.

2. Reduce $\frac{1}{7}$ to a decimal.

OPERATION.

```
7)1,0(,1428+ Ans.
  7
  --
  30
  28
  --
   20
   14
   --
    60
    56
    --
     4
```

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ to decimals. *Answers*, ,25. ,5. ,75.

4. Reduce $\frac{5}{26}$, $\frac{12}{480}$ and $\frac{9}{1129}$, to decimals. *Answers*, ,1923+,025,00797+

CASE 2.

To reduce numbers of different denominations, as of money, weight and measure, to their decimal values.

RULE.

" I. Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest."

"II. Oppofite to each dividend, on the left hand, place fuch a number for "a divifor as will bring it to the next fuperior denomination, and draw a "line perpendicularly between them.

"III. Begin with the higheft, and write the quotient of each divifion, as "decimal parts, on the right hand of the dividend next below it, and fo on, "till they are all ufed, and the laft quotient will be the decimal fought."

EXAMPLES.

1. Reduce 10*s*. 6¾*d*. to the fraction of a pound.

OPERATION.

4	3,
12	6,75
20	10,5625
	,528125 *Ans*.

The given numbers arranged for the operation, all ftand as integers. I then fuppofe 2 cyphers annexed to the 3 (3,00) which divided by 4, the quotient is 75, which I write againft 6 in the next line and the fum thus produced (6,75) I divide by 12, placing the quotient, (5625) at the right hand of the 10; laftly, I divide by 20 and the quotient, (,528125) is the decimal required.

2. Reduce 13s. 5¼d. to the decimal of a pound. *Ans*. ,6729+

3. Reduce 12pwts. 14grs. to the decimal of an ounce. *Ans*. ,6291.

CASE 3.

To find the value of any given decimal in the terms of an integer.

RULE.

MULTIPLY the decimal by that number, which it takes of the next lefs denomination to make *one* of that denomination in which the decimal is given, and cut off fo many figures for a remainder to the right hand of the quotient, as there are places in the given decimal. Proceed in the fame manner with the remainder, and continue to do fo thro' all the parts of the integer, and the feveral denominations ftanding on the left hand make the anfwer.

EXAMPLES.

1. What is the value of ,528125 of a pound?

OPERATION.

	,5 2 8 1 2 5
	2 0
Shillings,	1 0,5 6 2 5 0 0
	1 2
Pence,	6,7, 5 0 0 0 0
	4
Farthings,	3,0 0 0 0 0 0

Ans. 10s. 6¾d.

This question is the first example in the preceding case inverted, by which it will be seen, that questions in these two cases may reciprocally prove each other.

The given decimal being the decimal of a pound, and shillings being the next less inferior denomination, because 20 shillings make one pound, I multiply the decimal by 20, and cutting off from the right hand, of the product a number of figures, for a remainder, equal to the number of figures in the given decimal, leaves 10 on the left hand which are shillings. I then multiply the remainder which is the decimal of a shilling by 12, and cutting off as before, gives 6 on the left hand of pence; lastly, I multiply this last remainder, or decimal of a penny by 4 and find it to be 3 farthings, without any remainder. It then appears that ,528125 of a pound is in value 10s. 6¾d.

2. What is the value of ,73968 of a pound? *Ans.* £14 9½d

3. What is the value of ,768 of a pound Troy? *Ans.* 9*oz.* 4*pwt.*7$\frac{17}{25}$**grs.*

* $\frac{17}{25}$ *is the last remainder,* 680 *reduced to its lowest terms. A fraction is said to be reduced to its lowest terms, when there is no number which will divide both the numerator and denominator without a remainder. Thus, set to the fraction its proper denominator* $\frac{680}{1000}$, *then divide the numerator and the denominator by any number which will divide them both without a remainder, continue to do so as long as any number can be found that will divide them in that manner.*

$$Thus,\ 8)\ \frac{680}{1000} = \underbrace{\frac{85}{125}}^{5} = \frac{17}{25}$$

Supplement to Fractions.

QUESTIONS.

1. What are fractions ?
2. What are integers, or whole numbers ?
3. What are mixed numbers ?
4. Of how many kinds are fractions ?
5. How are Vulgar Fractions written ?
6. What is signified by the denominator of a fraction ?
7. What is signified by the numerator ?
8. How are Decimal Fractions written ?
9. How do Decimals differ from Vulgar Fractions ?
10. How can it be ascertained, what the denominator to a Decimal Fraction is, if it be not expressed ?
11. How do cyphers placed at the left hand of a Decimal Fraction affect its value ?
12. How are Decimals distinguished from whole numbers ?
13. In the addition of Decimals what is the rule for pointing off ?
14. What is the rule for pointing off Decimals in Subtraction ? In Multiplication ? and in Division ?
15. In what manner is the reduction of a vulgar Fraction to a decimal performed ?
16. How are numbers of different denominations, as pounds, shillings, pence, &c. reduced to their decimal values ?
17. If it be required to find the value of any given decimal in the terms of an integer what is the method of procedure ?

EXERCISES.

1. What is the sum of $79\frac{1}{2}$ $6\frac{1}{4}$ and of $\frac{3}{4}$ when added together ?

OPERATION.

```
 79,5
  6,25
   ,75
 ------
 86,50 Ans.
```

2. From 17 take $\frac{3}{4}$

OPERATION.

```
 17,
   ,75
 ------
 16,25 Remainder.
```

In CASE 1. *Ex. 3d,* under Reduction of decimal fractions the Scholar may notice, that $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ reduced to decimals are, ,25 ,5 and ,75. When numbers, therefore, for operation in either of the fundamental Rules, are incumbered with these fractions $\frac{1}{4}$,

3. Multiply $68\frac{1}{4}$ by $5\frac{1}{2}$

OPERATION.

```
  6 8,2 5
      5,5
  -------
  3 4 1 2 5
 3 4 1 2 5
 ---------
 3 7 5,3 7 5 Product.
```

4. Divide $26\frac{1}{4}$ by $5\frac{1}{2}$

OPERATION.

```
2,5)26,25(10,5 Quotient.
    25
    ---
     125
     125
     ---
```

$\frac{1}{2}$, $\frac{3}{4}$, substitute for them their equivalent decimal fractions, that is, for $\frac{1}{4}$,25 for $\frac{1}{2}$,5 for $\frac{3}{4}$,75 then proceed according to the rules already given for these respective operations in decimal fractions.

Many persons are perplexed by occurrences of a similar nature to the examples above. Hence it is seen in some measure the usefulness of Fractions, particularly decimal fractions. The only thing necessary to render any person adroit in these operations is to have riveted in his mind the rules for pointing as taught and explained in their proper places. They are not burthensome; every scholar should have them perfectly committed.

5 If a pile of wood be 18 feet long, $11\frac{1}{2}$ wide, and $7\frac{1}{4}$ high, how many cords does it contain?
Ans. 12 cords 68 feet 432 inches.*

A cord of wood is 128 solid feet; the proportions commonly assigned are, 8 feet in length, 4 in breadth, and 4 in height.

The contents of a load or pile of wood of any dimensions may be found by multiplying the length by the breadth and this product by the height; or, by multiplying the length, breadth, and height into each other. The last product divided by 128 will shew the number of cords, the remainder, if any, will be so many solid feet.

* *The 432 inches in the fraction ,25 of a foot valued according to* CASE 3, *Reduc. Dec.* FRACTIONS.

6. If a load of wood be 9 feet long, $3\frac{1}{2}$ feet wide, and 4 feet high, how many ſquare feet does it contain?

Ans. 126 *feet, which is 2 feet ſhort of a cord.*

7. What is the value ,725 of a day?

Ans. 17 *hours* 24 *minutes.*

8. What is the value of ,0625 of a ſhilling? *Ans.* 3 *farthings.*

9. Reduce 3Cwt. 0qrs. 7lb. 8oz. to the decimal of a Ton.

Ans. ,15334821+

10. Reduce 3 farthings to the decimal of a ſhilling. *Ans.* ,0625

11. Reduce $\frac{6}{480}$ to a decimal fraction. *Ans.* ,0125.

§ 3. Federal Money.

FEDERAL MONEY is the coin of the United States, eſtabliſhed by Congreſs, A. D. 1786. Of all coins this is the moſt ſimple, and the operations in it, the moſt eaſy.

The denominations are in a *decimal proportion*, as exhibited in the following

TABLE.

10 Mills	make one	Cent,
10 Cents	make one	Dime,
10 Dimes	make one	Dollar, *marked thus*, $
10 Dollars	make one	Eagle.

The expreſſion of any ſum in Federal Money is ſimply the expreſſion of a *mixed number* in decimal fractions. A dollar is the *Unit money* : dollars therefore muſt occupy the place of units, the leſs denominations, as dimes, cents, and mills, are decimal parts of a dollar, and may be diſtinguiſhed from dollars in the ſame way as any other decimals by a comma or ſeparatrix. All the figures to the left hand of dollars, or beyond Units place are eagles. Thus, 17 Eagles, 5 dollars, 3 dimes, 4 cents, and 6 mills are written—

Eag's ; or, Hundreds	*Tens.*	*Doll's ; or, Units.*	*Dimes ; or, Tenth parts.*	*Cents ; or, Hundredth parts.*	*Mills ; or, Thouſandth parts.*
1	7	5,	3	4	6

Of theſe, four are real coins, and one is imaginary.

The real coins are the Eagle, a gold coin ; the Dollar and the Dime, ſilver coins ; and the Cent, a copper coin. The mill is only imaginary, there being no piece of money of that denomination.

There are half-eagles, half-dollars, double-dimes, half-dimes, and half cents, real coins.

Theſe denominations, or different pieces of money, being in a tenfold proportion, conſequently, any ſum in Federal Money does of itſelf exhibit the particular number of each different piece of money contained in it. Thus 175,346 (*ſeventeen eagles, five dollars, three dimes, four cents, ſix mills*) contain 175346 mills, 17534$\frac{6}{10}$ cents, 1753$\frac{46}{100}$ dimes, 175$\frac{346}{1000}$ dollars, 17$\frac{5346}{10000}$ eagles. Therefore, eagles and dollars reckoned together, expreſs the number of dollars contained in the ſum ; the ſame of dimes and cents ; and this indeed is the uſual way of account, to reckon the whole ſum in dollars, cents and mills, thus,

Dolls.	*Cents.*	*Mills.*
$ 175	34	6

The Addition, Subtraction, Multiplication and Diviſion of Federal Money is performed in all reſpects as in Decimal Fractions, to which the Scholar is referred for the uſe of rules in theſe operations.

ADDITION OF FEDERAL MONEY.

1. Add 16 Eagles; 3 Eagles, 7 Dollars, 5 Cents; 26 Dollars, 6 Dimes, 4 Cents, 3 Mills; 75 Cents, 8 Mills; 40 Dollars, 9 Cents, together.

OPERATION.

Eag.	Dolls.	Dimes.	Cts.	Mills.
1 6	0,			
3	7,	0	5	
2	6,	6	4	3
	,	7	5	8
4	0,	0	9	
\$2 6	4,	5	4	1

Or, the sums may be reckoned in dollars, cents, and mills, thus,

Dolls.	Cts.	Mills.
160		
37	05	
26	64	3
	75	8
40	09	
\$264	54	1

2. If I am indebted 59 dollars, 112 dollars, 98 cts. 113 dolls. 15 cts. 15 dolls. 21 dolls. 50 cts. 200 dolls. 73 dolls. 35 dolls. 17 cts. 75 dolls. 20 dolls. 40 dolls. 33 cts. and 16 dolls. What is the fum which I owe?

Ans. \$781 13

Accountants generally omit the comma and diftinguifh *cents* from *dollars* by fetting them apart from the dollars.

SUBTRACTION OF FEDERAL MONEY.

1. FROM *Dolls.* 863,17
take, *Dolls.* 69,82

OPERATION.

```
  8 6 3,1 7
    6 9,8 2
  ---------
  7 9 3,3 5
```

Remainder, 7 9 3,3 5

2. FROM *Dolls.* 681
take, *Dolls.* 57,63
Remainder, Dolls. 623,37

MULTIPLICATION OF FEDERAL MONEY.

1. IF Flour be *Dolls.* 10,25 *per barrel,* what will 27 barrels cost?

OPERATION.

```
  1 0,2 5
      2 7
  -------
  7 1 7 5
2 0 5 0
---------
2 7 6, 7 5
```

Dolls. 2 7 6, 7 5 *Answer.*

POINT off the decimels in the product according to the rule in multiplication of decimals; if at any time there shall be more than three decimal figures, all beyond mills, or the third place, will be decimal parts of a mill.

2. MULTIPLY *Dolls.* 76,35
by *Dolls.* 37,46
Product, Dolls. 2860,0710

3. MULTIPLY *Dolls.* 24,675
by *Dolls.* 13,63.
Product, Dolls. 336,320 $\frac{25}{100}$

DIVISION OF FEDERAL MONEY.

1. IF 2728 bushels of wheat cost Dolls. 2961, how much is it per bushel?

OPERATION.

Bushels. Dolls. D. d. c. m.

```
2723)2961(1, 0 8 5 Answer.
     2728
     -----
      23300
      21824
      -----
      14760
      13640
      -----
       1120
```

WHEN the dividend consists of dollars only, if there be a remainder after division, cyphers must be annexed as in division of decimals.

2. DIVIDE *Dolls.* 3756 equally among 13 men; what will each man receive?
Ans. Dolls. 288,923.

3. DIVIDE *Dolls.* 16,75 by 27
Produƈt, 62 *Cents.*

REDUCTION OF FEDERAL MONEY.

CASE 1.

To reduce Pounds, Shillings, Pence and Farthings, to Dollars, Cents and Mills.

RULE.

SET down the pounds and to the right hand write half the greateſt even number of the given ſhillings; then conſider how many farthings there are contained in the given pence and farthings, and if the ſum exceed 12, increaſe it by 1, or if it exceed 36 increaſe it by 2, which ſum ſet down to the right hand of half the greateſt even number of ſhillings before written, remembering to increaſe the ſecond place, or the place next to ſhillings by 5, if the ſhillings be an odd number; to the whole ſum thus produced, annex a cypher and divide the ſum by 3; cut off the three right hand figures in the quotient, which will be cents and mills, the reſt will be dollars.

EXAMPLES.

1. REDUCE £ 47 7*s.* 10¾*d.* to Dollars, Cents and Mills.

OPERATION.

The Pounds.
Half of the number of ſhillings.
The farthings in pence and farthings increaſed according to rule.
Cypher annexed.

DIVIDE by 3) 4 7 3 9 5 0

Dolls. 1 5 7,9 8 3

IN this example to the right hand of pounds (47) I write 3, half the greateſt even number of the given ſhillings (7,) the farthings in 10¾*d* (43) increaſed by two (45) becauſe exceeding 36 and the ſecond place increaſed by 5 becauſe ſhillings were an odd number, are 95, which ſum written to the right hand of the 3, a cypher annexed, and the ſum divided by 3, gives the *Anſwer*, 157 *dollars*, 98 *cents, and* 3 *mills.*

If pounds only are given to be reduced, a cypher must be annexed and the number divided by 3; the quotient will be dollars. If there be a remainder annex more cyphers, and divide, the quotient will be cents and mills.

When there are no shillings, or only 1 shilling in the given sum, so there be no even number, write a cypher in place of half the even number of shillings, then proceed with the pence and farthings as in other cases.

If it be required to reduce pounds, shillings, pence, &c. to Dollars and Cents only, the cypher must not be annexed; in this case two figures only must be cut off from the quotient.

A little practice will make these operations extremely easy.

2. In £763 how many dollars, cents and mills?

Ans. 2543 *Dolls.* 33 *cents* 3 *m.*

3. In £17 1*s.* 6½*d.* how many dollars, cents and mills?

Ans. Dolls. 56 92 3.

4. In £109 3*s.* 8*d.* how many Dollars and cents?

Ans. Dolls. 363,944.+

5. In £86 6*s.* 5¼*d.* how many dollars, cents and mills?

Ans. Dolls. 287,74.

CASE 2.

To reduce Dollars, Cents, and Mills, to Pounds, Shillings, Pence, and Farthings.

RULE.

Multiply the given sum by 3, cut off the four right hand figures, which will be decimals of a pound, the left hand figures will be the pounds. To find the value of the decimals, double the first figure for shillings, and if the figure in the second place be 5, add another shilling, then call the figures in the second and third places, after deducting the 5 in the second place, so many farthings, abating 1 when they are above 12, and 2, when they are above 36.

EXAMPLES.

1. REDUCE 255 dollars 40 cents 6 mills, to pounds, shillings pence and farthings.

OPERATION.

255406
3
———
76|6218
£76 12s. 5d. *Answer*.

In this example having multiplied the given sum by 3 and cut off the four right hand figures of the product, I double the first figure (6) for shillings, the figures in the second and third places (21) abating 1 for being over 12, (20) I consider as farthings, equal to 5*d*. In *dolls*. 255,406 therefore, are £76 12*s*. 5*d*.

Here 8 in the fourth place of decimals ($\frac{8}{10000}$) *of a pound* being of inferior value is not reckoned. The loss in this place is always less than one farthing.

If there be no mills in the given sum, multiply as before and cut off 3 *figures* only.

If there be neither cents nor mills that is, *if the given sum be dollars*, multiply by 3 and cut off one figure only.

2. In *Dolls*. 392,75 how many pounds shillings, pence and farthings?
Ans. £117 16*s*. 6*d*.

3. In *Dolls*. 39,635 how many pounds, shillings, pence, &c.?
Ans. £11 17*s*. 9½*d*.

4. Reduce 134 Dollars, 65 Cents to pounds, shillings, pence and farthings.
Ans. £40 7*s*. 10¾*d*.

5. Reduce Dolls. 684 to pounds and shillings.
Ans. £205 4*s*.

Supplement to Federal Money.

QUESTIONS.

1. What is Federal Money? When was its eſtabliſhment, and by what authority?
2. What are the denominations in Federal Money?
3. Which is the unit Money?
4. How are dollars diſtinguiſhed from dimes, cents and mills?
5. What places do the different denominations occupy, from the decimal points?
6. How is the Addition of Federal Money Performed? Subtraction? Multiplication? Diviſion?
7. By what method are Pounds, Shillings, Pence ond Farthings reduced to Federal Money?
8. How are Dollars, Dimes, Cents and Mills, reduced to Pounds, Shillings, Pence and Farthings?

EXERCISES.

1. A man dies leaving an eſtate of 71600 Dollars, there are demands againſt the eſtate of Dolls. 39876,74; the reſidue is to be divided between 7 ſons; what will each one receive?
Ans. 4531 *Dolls.* 89 *cts.*

2. A man ſells 1225 buſhels of wheat at Dolls. 1,33 per buſhel, and receives Dolls. 93,76 for tranſportation; what does he receive in the whole?
Ans. Dolls. 1723,01.

3. Reduce £375 1*s*. 6 *d*. to dollars and cents.

Ans. Dolls. 1250,256

4. In £7 13*s*. 8*d*. how many dollars cents and mills?

Ans. Dolls. 25 61

5. Reduce Dolls. 781,27 to pounds, ſhillings, pence and farthings.

Ans. £234 7*s*. 7*d*.

6. Reduce Dolls. 98,763 to pounds, ſhillings, pence and farthings.

Ans. £29 12*s*. 7*d*.

TABLE

For reducing Shillings and Pence to Cents and Mills.

	0		Shill. 1		Shill. 2		Shill. 3		Shill. 4		Shill. 5	
pence	cts	Mills	cts	Mills	cts	Mills	cts	Mills	cts	Mills	cts	Mills
0			16	7	33	3	50		66	7	83	3
1	1	4	18	1	34	7	51	4	68	1	84	7
2	2	8	19	5	36	1	52	8	69	5	86	1
3	4	2	20	9	37	5	54	2	70	9	87	5
4	5	6	22	3	38	9	55	6	72	3	88	9
5	7		23	7	40	3	57		73	7	90	3
6	8	3	25		41	1	58	3	75		91	6
7	9	7	26	4	43		59	7	76	4	93	
8	11	1	27	8	44	4	61	1	77	8	94	4
9	12	5	29	2	45	8	62	5	79	2	95	8
10	13	9	30	6	47	2	63	9	80	6	97	2
11	15	3	32		48	6	65	3	82		98	6

To find by this Table the Cents and Mills in any ſum of Shillings and Pence under one Dollar, look the Shillings at top, and the Pence in the left hand column; then under the former and on a line with the latter, will be found the Cents and Mills ſought.

TABLE

For reducing the currencies of the several United States to Federal Money.

		N. Hamp. Mass. Rh. Island. Conn. and Virginia.	N. York, and N. Carolina.	N. Jersey, Pennsylvania, Delaware and Maryland.	S. Carolina, and Georgia.
		D.cts.m.	D.cts.m.	D.cts.m.	D.cts.m.
Farthings	1	, 3	, 3	, 3	, 4
	2	, 7	, 5	, 6	, 9
	3	, 10	, 8	, 8	, 14
Pence	1	, 14	, 10	, 11	, 18
	2	, 28	, 21	, 22	, 36
	3	, 42	, 31	, 33	, 54
	4	, 56	, 42	, 44	, 71
	5	, 69	, 52	, 56	, 89
	6	, 83	, 62	, 67	, 107
	7	, 97	, 73	, 78	, 125
	8	, 111	, 83	, 89	, 143
	9	, 125	, 94	, 100	, 161
	10	, 139	, 104	, 111	, 179
	11	, 153	, 114	, 122	, 196
Shillings.	1	, 167	, 125	, 133	, 214
	2	, 333	, 250	, 267	, 429
	3	, 500	, 375	, 400	, 643
	4	, 666	, 500	, 533	, 857
	5	, 833	, 625	, 667	1, 171
	6	1,000	, 750	, 800	1, 286
	7	1, 167	, 875	, 933	1, 500
	8	1, 333	1, 000	1, 067	1, 714
	9	1, 500	1, 125	1, 200	1, 929
	10	1, 667	1, 250	1, 333	2, 143
	11	1, 833	1, 375	1, 467	2, 657
	12	2, 000	1, 500	1, 600	2, 571
	13	2, 167	1, 625	1, 733	2, 786
	14	2, 333	1, 750	1, 867	3, 000
	15	2, 500	1, 875	2, 000	3, 214
	16	2, 667	2, 000	2, 133	3, 424
	17	2, 833	2, 125	2, 267	3, 643
	18	3, 000	2, 250	2, 400	3, 857
	19	3, 167	2, 375	2, 533	4, 071

TABLE
For reducing the currencies, &c. continued.

£.	New-Hamp. &c. &c. D. c.m.	New-York, &c. D.c.m.	New-Jersey, &c. &c. D.c.m.	South-Caroli. &c. D.c.m.
1	3,333	2,5	2,666	4,286
2	6,667	5,0	5,333	8,571
3	10,000	7,5	8,000	12,857
4	13,333	10,0	10,667	17,143
5	16,667	12,5	13,333	21,429
6	20,000	15,0	16,000	25,714
7	23,333	17,5	18,667	30,000
8	26,667	20,0	21,333	34,286
9	30,000	22,5	24,000	38,571
10	33,333	25,0	26,667	42,857
20	66,667	50,0	53,333	85,714
30	100,000	75,0	80,000	128,571
40	133,333	100,0	106,667	171,429
50	166,667	125,0	133,333	214,286
60	200,000	150,	160,000	257,143
70	233,333	175,	186,667	300,000
80	266,667	200,	213,333	342,857
90	300,000	225,	240,000	385,714
100	333,333	250,	266,667	428,571
200	666,667	500,	533,333	857,143
300	1000,000	750,	800,000	1285,714
400	1333,333	1000,	1066,667	1714,286
500	1666,667	1250,	1333,333	2142,857
600	2000,000	1500,	1600,000	2571,429
700	2333,333	1750,	1866,667	3000,000
800	2666,666	2000,	2133,333	3428,571
900	3000,000	2250,	2400,000	3857,143
1000	3333,333	2500,	2666,667	4285,714

TABLE
For reducing Federal Money to the currencies of the ſeveral United States.

D. Cts.	New-Hamp. &c. &c. Dol. 6*s*. £. s. d. q.	New-York, &c. Dol. 8*s*. £. s. d. q.	New-Jerſey, &c. &c. Dol. 7*s*6 £. s. d. q.	South-Caroli. &c. Dol. 4*s*8 £. s.d. q.
,01	3	1 0	1 0	2
,02	1 2	2 0	1 3	1 0
,03	2 1	3 0	2 3	1 3
,04	3 0	3 3	3 2	2 1
,05	3 2	4 3	4 2	2 3
,06	4 1	5 3	5 2	3 1
,07	5 0	6 3	6 1	4 0
,08	5 3	7 3	7 1	4 2
,09	6 2	8 3	8 0	5 0
,10	7 1	9 2	9 0	5 2

TABLE

For reducing the currencies, &c. continued.

	New-Hamp. &c. &c.				New-York, &c.				New-Jersey, &c. &c.				South-Carolina. &c.			
Dolls. cts.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.	£.	s.	d.	q.
,20		1	2	2		1	7	1		1	6	0			11	1
,30		1	9	2		2	4	3		2	3	0		1	4	3
,40		2	4	3		3	2	2		3	0	0		1	10	2
,50		3	0	0		4	0	0		3	9	0		2	4	0
,60		3	7	1		4	9	2		4	6	0		2	9	2
,70		4	2	2		5	7	1		5	3	0		3	3	1
,80		4	9	2		6	4	3		6	0	0		3	8	3
,90		5	4	3		7	2	2		6	9	0		4	2	2
1,		6	0	0		8	0	0		7	6	0		4	8	0
2,		12	0	0		16	0	0		15	0	0		9	4	0
3,		18	0	0	1	4	0	0	1	2	6	0		14	0	0
4,	1	4	0	0	1	12	0	0	1	10	0	0		18	8	0
5,	1	10	0	0	2	0	0	0	1	17	6	0	1	3	4	0
6,	1	16	0	0	2	8	0	0	2	5	0	0	1	8	0	0
7,	2	2	0	0	2	16	0	0	2	12	6	0	1	12	8	0
8,	2	8	0	0	3	4	0	0	3	0	0	0	1	17	4	0
9,	2	14	0	0	3	12	0	0	3	7	6		2	2	0	0
10,	3	0	0	0	4	0	0	0	3	15	0		2	6	8	0
20	6				8				7	10	0		4	13	4	
30	9				12				11	5	0		7	0	0	
40	12				16				15	0	0		9	6	8	
50	15				20				18	15	0		11	13	4	
60	18				24				22	10	0		14	0	0	
70	21				28				26	5	0		16	6	8	
80	24				32				30	0	0		18	13	4	
90	27				36				33	15	0		21	0	0	
100	30				40				37	10	0		23	6	8	
200	60				80				75	0	0		41	13	4	
300	90				120				112	10	0		70	6	0	
400	120				160				150	0	0		93	6	8	
500	150				200				187	10	0		116	13	4	
600	180				240				225	0	0		140	0	0	
700	210				280				262	10	0		163	6	8	
800	240				320				300	0	0		186	13	4	
900	270				360				337	10	0		210	0	0	
1000	300				400				375	0	0		233	6	8	
2000	600				800				750				466	13	4	
3000	900				1200				1125				700	0	0	
4000	1200				1600				1500				933	6	8	
5000	1500				2000				1875				1166	13	4	
6000	1800				2400				2250				1400	0	0	
7000	2100				2800				2625				1633	6	8	
8000	2400				3200				3000				1866	13	4	
9000	2700				3600				3375				2100	0	0	
10000	3000				4000				3750				2333	6	8	

§ 4. Interest.

INTEREST is the allowance given for the use of money, by the Borrower to the Lender. It is computed at so many dollars for each hundred lent for a year, *(per annum)* and a like proportion for a greater or less time. The highest rate is limited by our laws to 6 *per cent.* that is 6 dollars for a hundred dollars, 6 cents for a hundred cents, £6 for a £100, &c. This is called *legal interest*, and is always understood when no other rate is mentioned.

There are three things to be noticed in Interest.

1. The PRINCIPAL ; or, money lent.
2. The RATE ; or, sum *per cent.* agreed on.
3. The AMOUNT ; or, principal and Interest added together.

Interest is of two sorts, *simple* and *compound.*

1. Simple Interest is that which is allowed for the principal only.
2. Compound Interest is that which arises from the interest being added to the principal and (continuing in the hands of the lender) becomes a part of the principal, at the end of each stated time of payment.

GENERAL RULE.

1. *For one year,* multiply the principal by the rate, from the product cut off the two right hand figures of the dollars, which will be cents, those to the left hand will be dollars ; or which is the same thing, remove the *separatrix,* from its natural place two figures towards the left hand, then all those figures to the left hand will be dollars, and those to the right hand will be cents, mills, and parts of a mill.

In the same way is calculated the interest on any sum of money in pounds, shillings, pence and farthings, with this difference only, that the two figures cut off to the right hand of pounds, must be reduced to the lowest denomination, each time cutting off as at first.

2. *For two or more years,* multiply the interest of one year by the number of years.

3. *For months,* take proportional or aliquot parts of the interest for 1 year, that is, for 6 months, $\frac{1}{2}$; for 4 months $\frac{1}{3}$; for 3 months, $\frac{1}{4}$, &c.

For days, the proportional or aliquot parts of the interest for 1 month, allowing 30 days to a month.

EXAMPLES.

1. What is the interest of *Dolls* 86,446 for one year, at 6 *per cent ?*

OPERATION.

Dolls.	*cts.*	*m.*	
86,	44	6	*principal.*
		6	*rate.*
5| 18,	67	6	*Interest.*

In the product of the principal multiplied by the rate is found the answer. Thus, cutting off the two right hand figures from the dollars leave 5 on the left hand, which is dollars ; the two figures cut off (18) are cents, the next figure (6) is mills ; all the figures which may chance to be at the right hand of mills, are parts of a mill ; hence we collect the *Ans.* 5 *dolls.* 18 *cts.* 6$\frac{76}{100}$ *m.*

2. What is the interest of Dolls. 365 14 cents 6 mills, for three years, 7 months and 6 days?

OPERATION.

```
                  3 6 5, 1 4 6 principal.
                             6 rate.
                  -------------
6 Months 1/2)2  1| 9 0, 8 7 6 interest for one year.
                             3
              -------------
              6 5, 7 2 6 2 8 interest for 3 years.
1 Month 1/6)  1 0, 9 5 4 3 8 interest for 6 months.
  6 Days 1/5)  1, 8 2 5 7 3 interest for 1 month.
                 , 3 6 5 1 4 interest for 6 days.
              -------------
              7 8, 8 7 1 5 3 interest for 3 years, 7 months, and 6 days; that
                             is 78 dolls. 87cts. 1 53/100 m.
```

Because 7 months is not an even part of a year, take two such numbers as are even parts and which added together will make 7 (6 and 1,) 6 months is ½ of a year, therefore, for 6 months, divide the interest of one year by 2; again, 1 month is ⅙ of 6 months, therefore *for* 1 *month*, divide the interest of 6 months by 6. For the days, because 6 days is ⅕ of a month, or of 30 days, therefore, *for* 6 *days*, divide the interest of 1 month by 5. Lastly add the interest of all the parts of the time together, the sum is the answer.

3. What is the interest of £71 7*s*. 6½*d*. for 1 year, at 6 per cent.

OPERATION.

```
     £.  s.  d.  q.
     71   7   6   2
                  6
   ---------------
  £428    5   3   0
    20
   ----
s. 5|65
    12
   ----
d. 7|83
     4
   ----
q. 3|32 Ans. £4 5s. 7¾d.
```

4. What is the interest of 16*s*. 8*d*. for 1 year? *Ans*. 1 *shilling*.

When the rate is at 6 *per cent, there is not, perhaps, a more concise and easy way of casting interest, on any sum of money in Dollars, Cents, and Mills, than by the following*

METHOD.

Write down half the greatest *even number* of months for a multiplier; if there be an odd month it must be reckoned 30 days, for which and the given days, if any, seek how many times you can have 6 in the sum of them, place the figure for a decimal at the right hand of half the even number of months, already found, by which multiply the principal; observing in pointing off the product, to remove the decimal point or separatrix *two figures* from its natural place towards the left hand, that is point off *two* more places for decimals in the product, than there are decimal places in the multiplicand and multiplier counted together; then all the figures to the left hand of the point, will be dollars, and those to the right hand, dimes, cents, mills, &c. which will be the interest required.

Should there be a remainder in taking one sixth of the days, reduce it to a vulgar fraction, for which take aliquot parts of the multiplicand. Thus,

If the remainder be	$1=\frac{1}{6}$, divide the multiplicand	by 6
If - - - -	$2=\frac{1}{3}$, - - - -	by 3
If - - - -	$3=\frac{1}{2}$, - - - -	by 2
If - - - -	$4=\frac{1}{3}$, - - - -	by 3 twice.
If - - - -	$5=\frac{1}{2}$ and $\frac{1}{3}$ - - -	by 2 and 3

The quotients which in this way occur, must be added to the product of the principal multiplied by half the months, &c. the sum thus produced, will be the interest required.

When there are days, but a less number than 6, *so that* 6 *cannot be contained in them*, put a cypher in place of the decimal at the right hand of the months, then proceed in all respects as above directed.

NOTE. In casting interest, each month is reckoned 30 days.

EXAMPLES.

1. What is the interest of *Dolls.* 76,54 for 1 year 7 months, and 11 days?

OPERATION.

```
        7 6, 5 4
            9 6
      ----------
      4 5 9 2 4
    6 8 8 8 6
½       3 8 2 7
⅓       2 5 5 1
  --------------
Ans. 7, 4 1 1 6 2
Dolls. cts. m.
```

The number of months being 19, the greatest even number is 18, half of which is 9, which I write down; then seeking how often 6 is contained in 41, (the sum of the days in the odd month and given days) I find it will be 6 times, which I also set down at the right hand of half the even number of months for a decimal, by which together I multiply the principal. In taking one sixth of the days (41) there will be a remainder of $5=\frac{1}{2}$ and $\frac{1}{3}$ for which I take, first *one half* the multiplicand, that is, divide the multiplicand by 2, then by 3, and these quotients added, with the products of half the even number of months &c. the sum of them will shew the interest required, observing to count off *two* more figures for decimals in the product than there are decimal figures in both the multiplier and multiplicand, counted together.

For the conciseness and simplicity of the above METHOD, it is conceived, that Instructors will recommend it to their Pupils in preference to any other.

2. What is the intereſt of Dolls. 5,93 for 2 years and 8 months ?
Ans. 94 *cents,* 8 *mills.*

3. What is the intereſt of Dolls. 67,62 for 3 years and 2 months ?
Ans. 12 *Dolls.* 84 *cts.* 7 *mills.*

4. What is the intereſt of 91 cents for 27 years ?
Ans. 1 *dol.* 47*cts.* 4*m.*

When the intereſt on any ſum is required for a great number of years, it will be eaſier, firſt to find the intereſt for 1 year, then multiply the intereſt ſo found by the number of years.

5. What is the intereſt of Dolls. 2870,32 for 10 days ?
Ans. 4 *dolls.* 78*cts.* 3*m.*

When the rate is any other than 6 PER CENT, *firſt find the intereſt at* 6 *per cent, then divide the intereſt ſo found by ſuch parts as the intereſt at the rate required, exceeds or falls ſhort of the intereſt at* 6 *per cent, and the quotient added to or ſubtracted from the intereſt at* 6 *per cent, as the caſe may be, will give the intereſt at the rate required.*

6. What is the intereſt of Dolls. 137,84 for 2 years and 6 months at 5 per cent ? *Ans. Dolls.* 17,23.

7. What is the intereſt of Dolls. 79,07 for 10 months at 8 per cent ?
Ans. Dolls. 5,271.

8. What is the intereſt of Dolls. 2,29 for 1 month 19 days at 3 per cent? *Ans.* 9 *mills.*

9. What is the intereſt of Dolls. 18 for 2 years, 14 days at 7 per cent? *Ans.* 2 *dolls.* 56*cts.* 9*m.*

10. What is the intereſt of Dolls. 1600 for one year, and 3 months? *Ans.* 120 *dolls.*

11. What is the intereſt of Dolls. 5,811 for one year 11 months? *Ans.* 66 *cents.* 8*m.*

12. What is the intereſt of Dolls. 17,68 for 11 months, and 28 days? *Ans. dolls.* 1,054.

13. What is the intereſt of Dolls. 861,12 for 9 months, 25 days at 7 per cent? *Ans. dolls.* 49,394.

14. What is the intereſt of Dolls. 105,61 for 1 year, 7 months, and 6 days? *Ans.* 10 *dolls.* 13*cts.* 8*m.*

15. What is the intereſt of Dolls. 86 for 9 months? *Ans. dolls.* 3,87.

16. What is the intereſt of 78 Dolls. 36*cts.* for 5 years 10 months, and 3 days? *Ans.* 27 *dolls.* 46*cts.* 5*m.*

17. What is the intereſt of 812 Dolls. 30*cts.* for 2 years, 8 months, and 4 days? *Ans.* $130,509.

To this mode of computing intereſt, I would add from the "*Maſſachuſetts Juſtice,*" a

METHOD

Of computing the intereſt due on bonds, notes, &c. when partial payments may at different times be made, as eſtabliſhed by the Courts of Law in Maſſachuſetts.

RULE.

Caſt the intereſt up to the firſt payment, and if the payment exceed the intereſt, deduct the exceſs from the principal, and caſt the intereſt upon the remainder to the time of the ſecond payment. If the payment be leſs than the intereſt, place it by itſelf, and caſt on the intereſt to the time of the next payment, and ſo on, until the payments exceed the intereſt, then deduct the exceſs from the principal, and proceed as before.

EXAMPLES.

Suppoſe A ſhould have a bond againſt B for 1166 dollars, 66 cents, and 6 mills, dated May 1, 1796, upon which the following payments ſhould be made, viz.

	Dollars, Mills.	Months,	Days.
1. December 25, 1796, - - -	166,666	7	24
2. July 10, 1797, - - - - -	16,666	6	15
3. September 1, 1798, - - -	50,000	13	21
4. June 14, 1799, - - - - -	333,333	9	13
5. April 15, 1800, - - - - -	620,000	10	1
What will be due upon it Auguſt 3, 1801?		15	18

Ans. Dolls. 237 76 *cents.*

To facilitate the operation, let the ſpace of time from the date of the Bond to the day of the firſt payment, and from the time of one payment to that of another, and from that of the laſt payment to the time of ſettlement, be firſt computed and ſet down againſt the day of payment as above. Then ſet down

the ſum on which the intereſt is to be caſt, with the intereſt and payments in columns thus,

	Principal.	Time.		Intereſt.	Payments.	Exceſs.
	Dolls. Mills.	*Mo.*	*Da.*	*Dolls. M.*	*Dolls. M.*	*Dolls. M.*
1	1166,666	7	24	45,499	166,666	121,167
	121,167					
2	1045,499	6	15	33,978	16,666	
3	1045,499	13	21	71,616	50,000	
4	1045,499	9	13	49,312	333,333	
	245,093			154,906	399,999	245,093
5	800,406	10	1	40,153	620,000	579,847
	579,847					
	220,559	15	18	17,203		

The laſt remainder	220,559
Intereſt from the laſt payment	17,203
Sum due Aug. 1, 1801,	237,762

2. Suppoſing a note of 867 dollars, 33 cents, dated January 6, 1794, upon which the following payments ſhould be made, viz.

	Dolls. Cents.
1. April 16, 1797, - - -	136,44
2. April 16, 1799, - - -	319,
3. Jan. 1, 1800, - - -	518,68

What would be due July 11, 1801? *Ans. Dolls.* 215,103.

COMPOUND INTEREST,

Is calculated by adding the intereſt to the principal at the end of each year and making the amount the principal for the ſucceeding year ; then the given principal ſubtracted from the laſt amount, the remainder will be the compound intereſt.

A conciſe and eaſy method of caſting Compound Intereſt, at 6 per cent. on any ſum in Federal Money.

RULE.

Multiply the given ſum, if

For 2 *years by*	112,36	*For* 7 *years, by*	150,3630
3 *years* —	119,1016	8 *years* —	159,3848
4 *years* —	126,2476	9 *years* —	168,9478
5 *years* —	133,8225	10 *years* —	179,0847
6 *years* —	141,8519	11 *years* —	189,8298

NOTE 1. Three of the firſt higheſt decimals, in the above numbers, will be ſufficiently accurate for moſt operations ; the product, remembering to move the ſeparatrix *two* figures from its natural place towards the left hand will then ſhew the amount of principal and compound intereſt for the given number of years. Subtract the principal from the amount, and it will ſhew the compound intereſt.

2. *When there are months and days ;* firſt find the amount of principal and compound intereſt for the years, agreeable to the foregoing method, then, for the months and days caſt the ſimple intereſt on the amount thus found ; this added to the amount will give the anſwer.

3. Any ſum of money at Compound Intereſt, will double itſelf in 11 years 10 months and 22 days.

EXAMPLES.

1. What is the compound intereſt of $56 75 for 11 years ?

OPERATION.

56,75	
189,829	
51075	
11350	
45400	
61075	
45400	
5675	
$107,7279575	*Amount.*
56,75	*Principal ſubtracted.*
$50,97	*Compound intereſt.*

2. What is the amount of $236 at compound intereſt, for 4 years, 7 months and 6 days ?

OPERATION.

126,2476	
236	
7574856	
3787428	
2524952	
$297,944336	*Amount*
3,6	*for 4 years.*
1787664	
893832	
$10,725984	*Intereſt for 7 mo. 6 days.*
297,944	*Amount for 4 years added.*
$308,669	*Anſwer.*

Supplement to S. Interest.

QUESTIONS.

1. WHAT is Intereſt ?
2. What is underſtood by 6 per cent ? 3 per cent ? 8 per cent ? &c.
3. What per cent per annum is allowed by law to the lender for the uſe of his money ?
4. What is underſtood by the principal ? The rate ? The amount ?
5. Of how many kinds is intereſt ? In what does the difference conſiſt ?
6. How is ſimple intereſt calculated for one year, in Federal Money ?
7. For more years than one, how is the intereſt found ?
8. When there are months and days, what is the method of procedure ?
9. What other method is there of caſting intereſt on ſums in Federal Money ?
10. When the days are a leſs number than 6, ſo that 6 cannot be contained in them, what is to be done ?
11. How is ſimple intereſt caſt in pounds, ſhillings, pence and farthings ?
12. When partial payments are made at different times, how is the intereſt calculated ?

EXERCISES.

1. What is the intereſt of \$916,72, for one year and 4 months ?
Ans. \$73,327.

2. What is the intereſt of \$93,17 *cts.* for 11 days ?
Ans. 17*cts.*

3. What is the intereſt of \$5,19 for 7 months ?
Ans. 18 *cts.* 1 *m.*

4. What is the intereſt of \$1,07 for 3 years, 6 months and 15 days ?
Ans. 22 *cts.* 7 *m.*

5. What is the intereſt of £41 11*s.* $3\frac{1}{4}$*d.* for a year and 2 months?
Ans. £2 18 $2\frac{1}{4}$.

6. What is the intereſt of $273,51 at 7 per cent for 1 year and 10 days?
Ans. $19,677

7. Suppoſing a note of $317,92 dated July 5, 1797, on which were the following payments—Sept. 13, 1799, $208,04: March 10, 1800, $76; what was the ſum due Jan. 1, 1801? *Ans.* $83,991.

§ 5. Compound Multiplication.

COMPOUND MULTIPLICATION is when the Multiplicand consists of several denominations. It is particularly useful in finding the value of Goods.

The different denominations in what was formerly called *Lawful Money*, render this rule with some others in Arithmetic, as *Compound Division* and *Practice*, rules of great usefulness, quite tedious, and the variety of cases necessarily introduced, extremely burthensome to the memory. This *lumber of the mind* might be almost wholly dispensed with, were the habit of reckoning in *Federal Money* generally adopted through the U. States.

For important reasons, *Pounds, Shillings, Pence and Farthings* ought to fall wholly into disuse: Federal Money is our National currency; the scholar might encompass the most useful rules of Arithmetic in half the time; the value of commodities bought and sold, might be cast with half the trouble, and with much less liability to errors, were all the calculations in money universally made in *Dollars, Cents* and *Mills*. But this, to be practised must be taught; it must be taught in our schools, and so long as the prices of goods and almost every man's accounts are in *Pounds, Shillings, Pence and Farthings*, this mode of reckoning must not be left untaught.

To comprise the greater usefulness, and also to shew the great advantage which is gained by reckoning in Federal Money, I have contrasted the two modes of account, and in separate columns, on the same page, have put the same questions in *Old Lawful* and in *Federal Money*.

OPERATIONS.

In Pound, Shill. Pence, Farthings,

CASE 1.

When the quantity does not exceed 12 *yards, pounds, &c.* set down the price of 1 yard or pound, and place the quantity underneath the lowest denomination for a multiplier. Begin by multiplying the lowest denomination, and carry by the same rules from one denomination to another, as in Compound Addition.

EXAMPLES.

1. What will 7 yards of cloth cost at 9*s*5 per yard?

OPERATION.

	£.	s.	d.	
	0	9	5	*price of* 1 *yard.*
			7	*yards.*
Ans.	3	5	11	*price of* 9 *yards.*

I say, 7 times 5 is 35 pence=2*s*11. I set down 11 and carry 2, saying, 7 times 9 is 63, and 2 I carry is 65*s*.=£. 3 5*s*. which I set down.

In Dollars, Cents, Mills.

IN ALL CASES,

Multiply the price and the quantity together, according to the rules of multiplication in *Decimal Fractions*, and *Federal Money*, and the product will be the answer, That is,

Multiply as in simple multiplication, and from the product point off so many places for cents and mills as there are places of cents and mills in the price.

EXAMPLES.

1. What will 7 yards of cloth cost at \$1,57 *(equal to* 9*s*5*)* per yard?

OPERATION.

	D. cts.	
	1, 57	*Price.*
	7	*quantity.*
Ans.	10,99	*price of* 9*y*.

As there are two decimal places in the price, so I make two in the product.

Pounds, Shill. Pence, Farthings.

2. What will 9 pounds of ſugar coſt at 10*d*. per pound? *Ans.* 7*s*6.

3. What will 6 yards of cloth coſt at £1 10*s*. 5*d*. per yard? *Ans.* £9 2*s*. 6*d*.

CASE 2.

When the quantity exceeds 12 *and is any number within the Multiplication Table* multiply by two ſuch numbers, as when multiplied together, will produce the given quantity.

If no two numbers will do this exactly, multiply by two ſuch numbers as come the neareſt to it, and by the deficiency or exceſs, multiply the multiplicand, and this product added to, or ſubtracted from the firſt product, as the caſe may require, gives the anſwer.

EXAMPLES.

1. What will 42 yards of cloth coſt at 15*s*9 per yard?

OPERATION.

	£.	s.	d.	
	0	15	9	*price* 1 *yard.*
Multiplied by			6	
	4	14	6	*price of* 6*yds.*
Multiplied by			7	
Ans.	33	1	6	*price of* 42*yds.*

Becauſe 6 times 7 is 42, I multiply the price of 1 yard by 6, and this product by 7, as the rule directs.

Dollars, Cents, Mills.

2. What will 9 pounds of ſugar coſt at $0, 139 per pound? *Ans.* $1,251

3. What will 6 yards of cloth coſt at $5,07 per yard? *Ans.* $30,42

4. What will 42 yards of cloth coſt at $2,625 per yard?

OPERATION.

	D.	cts.	m.
	2,	6 2 5	
		4 2	
	5	2 5 0	
	1 0 5	0 0	
	$1 1 0,	2 5 0	*Anſwer.*

Pounds, Shill. Pence, Farthings.

2. What will 125 yards of cloth coſt at 5s7 per yard ? *Ans.* £34 17 11.

3. What will 51 pounds of tea coſt at 3s6 per pound ? *Ans.* £8 18s. 6d.

4. What will 130 yards of cloth coſt at £2 3s. 9d. per yard ? *Ans.* £284 7s. 6d.

CASE 3.

When the multiplier, that is, the quantity exceeds 144, multiply firſt by 10, and this product again by 10, which will give the price of 100 yards, &c. and if the quantity be even hundreds, multiply the price of one hundred by the number of hundreds in the queſtion, and the product will be the anſwer ; if there are odd numbers, multiply the price of 10 by the number of tens, and the price of unity, or 1, by the number of units, then theſe ſeveral products added together will be the anſwer.

Dollars, Cents, Mills.

5. What will 125 yards of cloth coſt at 93 cents per yard ? *Ans.* $116,25

6. What will 51 pounds of tea coſt at $0,583 per pound ? *Ans.* $29,733

7. What will 130 yards of cloth coſt at $7,25 per yard ? *Ans.* $942,50

Pounds, Shill. Pence, Farthings.

EXAMPLES.

1. What will 563 yards of cloth cost at £1 6s. 7d. per yard ?

OPERATION.

	£.	s.	d.	
	1	6	7	*price of* 1 *yard.*
			10	
	13	5	10	*price of* 10 *yds.*
			10	
	132	18	4	*price of* 100 *yds.*
			5	
	664	11	8	*price of* 500 *yds.*
6 *times* 10 *yds.*	79	15	0	*price of* 60 *yds.*
3 *times* 1 *yd.*	3	19	9	*price of* 3 *yds.*
Ans.	748	6	5	*price of* 563 *yds.*

2. What will 328 yards of cloth cost at 10s. 6½ per yard ?

Ans. £172 17 8d.

3. What will 624 yards of cloth cost at 12s. 8d. per yard ?

Ans. £395 4s.

Dollars, Cents, Mills.

8. What will 563 yards of cloth cost at $4,43 per yard ?

OPERATION.

		Yds.	5	6	3	
		$	4,	4	3	
		1	6	8	9	
	2	2	5	2		
2	2	5	2			
$2	4	9	4,	0	9	*Ans.*

9. What will 328 yards of cloth cost at $1,757 per yard ?

Ans. $576,296

10. What will 624 yards of cloth cost at $2,111 per yard ?

Ans. $1317,264

Supplement to Compound Multiplication.

QUESTIONS.

1. WHAT is Compound Multiplication?
2. What is its uſe ?
3. Are operations moſt eaſy in OLD LAWFUL, or in FEDERAL MONEY ?
4. What is the rule for Compound Multiplication ?
5. When the quantity, that is, the Multiplier, exceeds 12, and is within the Multiplication Table, what are the ſteps to be taken ?
6. When no two numbers multiplied together will produce the given quantity, what then is to be done ?
7. When the multiplier exceeds 144, what is the method of procedure ?
8. When the price of goods is given in Federal Money, what is the general and univerſal rule for finding their value by multiplication ?

EXERCISES.

1. A man has 38 ſilver cups, each one weighing 1*oz.* 3*pwts.* 16*grs.* how much ſilver do they all contain ?
Ans. 3*lb.* *oz.*8. 19*pwts.* 8*grs.*

2. If a man travel 34 miles, 3 furlongs, and 17 rods in one day, how far will he travel in 62 days ?
Ans. 2134 *miles*, 4 *fur.* 14 *rods.*

3. What will 285 yards of cloth come to at £1 2*s.* 5¼*d.* per yard ?
Ans. £263 17*s.* 8½*d.*

4. If a horſe run a mile in 12 minutes, 16 ſeconds, in what time would he go 176 miles ?
Ans. 1*D.* 11*h.* 58*m.* 56*s.*

§ 6. Compound Division.

COMPOUND DIVISION is the dividing of different denominations.

OPERATIONS.

In Pounds, Shil. Pence, Farthings.

CASE I.

1. *When the Divisor, that is the quantity, does not exceed* 12, begin at the highest denomination, and in the manner of short division, find how many times the divisor is contained in it; place the quotient under its own denomination, and if any thing remain, reduce it to the next less denomination, and divide as before; so proceed through all the denominations.

2. *If the quantity exceeds* 12, *and there be any two numbers, which multiplied together will produce it*, divide the price first by one of those numbers, and this quotient by the other.

EXAMPLES.

1. If 5 yards of cloth cost £3 13*s* 6*d*. what is that per yard?

OPERATION.

	£.	s.	d.	
5)	3	13	6	*price of 5 yards.*
	0	14	$8\frac{1}{4}$	*price of 1 yard.*

Finding I cannot have the divisor (5) in the first denomination (£3) I reduce it to shillings, (60) and add in the 13 shillings, which make 73 shillings, in which the divisor (5) is contained 14 times, and 3 remain; I set down the 14, and the remainder (3 shillings) reduce to pence (36) and the 6*d*. added make 42 pence, in which the divisor is contained 8 times and *two* remain; I set down the 8, and reduce the 2 pence to farthings (8) in which I have the divisor *once* (1*qr*. or $\frac{1}{4}$*d*.) and a remainder of $\frac{3}{5}$ of a farthing, which being of small value is neglected.

2. If 48 yards of cloth cost £4 16*s*. $4\frac{1}{2}$*d*. what is that per yard?

Ans. £0 2*s*.

In Dollars, Cents, Mills.

IN ALL CASES,

Divide the price by the quantity, and point off so many places for cents and Mills in the product as there are places of cts. and mills in the dividend.

If the quantity be a composite number, that is, produced by the multiplication of two numbers, the operation may be varied by dividing the price first by one of those numbers, and this quotient by the other.

EXAMPLES.

1. If 5 yards of cloth cost $12,25 what is that per yard?

OPERATION.

	D.	Cts.
5)	1 2,	2 5
Ans.	2,	4 5

There are two decimal places in the dividend. I, therefore, point off two places for decimals, or cents in the quotient.

2. If 48 yards of cloth cost $16,06 what is that per yard?

Ans. $0,33

Pounds, Shill. Pence, Farthings.

3. If 24lb. of tea cost £2 7s. 9½d. what is that per lb.?

Ans. £0 1s. 11¼d.

4. If 35 yards of cloth cost £42 6s. 7½d. what is that per yard?

Ans. £1 4s. 2¼d.

CASE 2.

1. "*Having the price of an hundred weight* (112lb.) *to find the price of* 1lb. divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1lb. required."

2. *If the number of hundred weight be more than one,* first divide the whole price by the number of hundreds, then proceed as before.

EXAMPLES.

1. If 1cwt. of sugar cost £3 7s. 6d. what is that per lb.?

OPERATION.

	£	s.	d.	q.	
8)	3	7	6		*price of* 1 *cwt.*
7)	0	8	5	1	*price of* 14*lb.* or $\frac{1}{8}$ *cwt.*
2)		1	2	2	*price of* 2*lb.* or $\frac{1}{56}$ *cwt.*
Ans.	0		7	1	*price of* 1*lb.*

Dollars, Cents, Mills.

3. If 24 lb. of tea cost \$7,97 what is that per lb.? *Ans.* \$0,332.

4. If 35 yards of cloth cost \$141,103 what is that per yard?

Ans. \$4,031.

The same may be done in Federal Money.

5. If 1cwt. of sugar cost \$11,[illegible] *cts.* what is that per lb.

Ans. 10 *cts.*

Pounds, Shill. Pence, Farthings.

2. If 8 cwt. of cocoa cost £15 7*s.* 4*d.* what is that per lb.? *Ans.* 4*d.*

3. If 3 cwt. of sugar cost £15 13*s.* what is that per lb.? *Ans.* 11*d.*

Dollars, Cents, Mills.

6. If 8 cwt. of cocoa cost $51,223 what is that per lb.? *Ans.* 5 *cents*, 7 *mills.*

7. If 3 cwt. of sugar cost $52,167 what is that per lb.? *Ans.* 15 *cents* 5 *mills.*

CASE 3.

"*When the divisor is such a number as cannot be produced by the multiplication of small numbers*, divide after the manner of long division, setting down the work of dividing and reducing."

Pound, Shill. Pence, Farthings.

EXAMPLES.

1. If 46 yards of cloth coſt £53 10*s*. 6*d*. what is that per yard?

OPERATION.

```
    £  s. d.  £. s. d.
46)53 10 6(1  3  3¼d. Ans.
   46
   --
    7
   20
  ---
46)150(3
   138
   ---
    12
    12
   ---
46)150(3
   138
   ---
    12
     4
   ---
46)48(1
   46
   --
    2
```

2. If 263 buſhels of wheat coſt £86 7*s*. 10*d*. what is that per buſhel?

Ans. 6*s*. 6½*d*.

3. If 670 gallons of wine coſt £147 1*s*. 11*d*. what is that per gallon?

Ans. 4*s*. 4½*d*.

Dollars, Cents, Mills.

8. If 46 yards of cloth coſt $178,416 what is that per yard?

Ans. $3,878.

9. If 263 buſhels of wheat coſt $287,973, what is that per buſhel?

Ans. 1,095.

10. If 670 gallons of wine coſt $490,32; what is that per gallon?

Ans. $0,73.

Supplement to Compound Division.

QUESTIONS.

1. What is Compound Divifion ?
2. When the price of any quantity, not exceeding 12, of yards, pounds, &c. is given in pounds, fhillings, pence and farthings, how is the price of one yard found ?
3. When the quantity is fuch a number as cannot be produced by the multiplication of fmall numbers, what is the method of procedure ?
4. Having the price of an hundred weight given, in what way is found the price of 1 lb. ?
5. If there be feveral hundred weight, what are the fteps of operating ?
6. When the price is given in Federal Money, what is the method of operating ?

EXERCISES.

Pounds, Shill. Pence, Farthings.

1. If 10 fheep coft £4 5*s*. 7*d*. what is the price of each ? *Ans*. 8*s*. $6\frac{1}{2}$.

2. If 84 cows coft £253 13*s*. what is the price of each ?

Ans. £3, $0\,4\frac{1}{2}$.

Dollars, Cents, Mills.

Let the fcholar reduce the price of fheep and of the cows to Federal Money, and perform the operations in Dolls. Cents and Mills.

Price of 1 *fheep*, \$1,426.

Price of 1 *cow*, 10,065.

3. If 121 pieces of cloth measure 2896 yards 1qr. 3na. what does each piece measure ?

Ans. 23 *yards,* 3*qr.* 3*na.*

4. If 66 tea-ſpoons weigh 2lb. 10*oz.* 14pwt. what is the weight of each ?

Ans. 10*pwt.* $12\frac{1}{3}\frac{2}{33}$*grs.*

5. If 2 cwt. of rice coſt £2 11*s.* $6\frac{1}{2}$*d* what is that per lb. ? *Ans.* $2\frac{3}{4}$*d.*

6. At £2 11*s.* $6\frac{1}{2}$*d.* for 2 cwt. of rice, what is that in *Federal Money,* and what is that per lb. ?

Price of 1 *lb.* 3*cts.* 8*m.*

7. If 47 bags of Indigo weigh 12 cwt. 1qr. 26lb. 4oz. what does each weigh ? *Ans.* 1*qr.* 1*lb.* 12*oz.*

8. If 8 horſes eat 900 buſhels, and 1 peck of oats in 1 year, how much will each horſe eat per day ?

Ans. 1 *peck,* 1*qr.* 1*pt.* 2*gills.*

9. Divide £297 2*s.* 3*d.* among 4 men, 6 boys, and give each man 3 times so much as one boy; what will each man share, and each boy?

OPERATION.

The men have triple shares, therefore, multiply the number of men (4) by 3, and add the number of boys, (6) for a divisor.

	£.	s.	d.	£.	s.	d.	q.	
18)	297	2	3	(16	10	1	2 = 1	*boy's share.*
	18						3	
	117		*Ans.*	49	10	4	2 = 1	*man's share.*
	108							
	9							
	20							
	)182(10							
	18							
	2							
	12							
	)27(1							
	18							
	9							
	4							
	)36(2							
	36							

men.	*boys.*
4 and	6
3	
12	
6	
18 *the number of equal shares in the whole* = Divisor.	

PROOF.

£.	s.	d.	q.	
49	10	4	2	
			4	
198	1	6	0	*men's share.*
16	10	1	2 and	
			6	
99	0	9	0	*boy's share.*
£.297	2	3	0	*added.*

10. Divide £.39 12*s.* 5*d.* among 4 men, 6 women, and 9 boys; give each man double to a woman, each woman double to a boy.

	£.	s.	d.	
Answer.	1	1	5	*a boy's share.*
	2	2	10	*a woman's share.*
	4	5	8	*a man's share.*

§ 7. Single Rule of Three.

THE Single Rule of Three, sometimes called the RULE *of* PROPORTION, is known by having *three terms* given to find the *fourth.*

It is of TWO kinds, *Direct* and *Indirect*, or *Inverse.*

SINGLE RULE OF THREE DIRECT.

The Single Rule of Three Direct teaches, by having three numbers given to find a fourth, which shall bear the same proportion to the third that the second does to the first.

It is evident that the value, weight, and measure of any commodity is proportionate to its quantity, that the amount of work, or consumption is proportionate to the time ; that gain, loss, and interest when the time is fixed, is proportionate to the capital sum from which it arises ; and that the effect produced by any cause is proportionate to the extent of that cause.

These are cases in direct proportion, and all others may be known to be so, when the number sought increases or diminishes along with the term from which it is derived. Therefore,

If *more* require *more*, or *less* require *less*, the question is always known to belong to the Rule of Three Direct.

More requiring more, is when the third term is greater than the first and requires the fourth term to be greater than the second.

Less requiring less, is when the third term is less than the first and requires the fourth term to be less than the second.

RULE.

"1. State the question by making that number which asks the question, "the third term, or putting it in the third place ; that which is of the same "name or quality as the demand, the first term, and that, which is of the "same name or quality with the answer required, the second term."

"2. Multiply the second and third terms together, divide by the first, and "the quotient will be the answer to the question, which (as also the remain- "der) will be in the same denomination in which you left the second term, "and may be brought into any other denomination required."

The chief difficulty that occurs in the *Rule of Three,* is the right placing of the numbers, or stating of the question ; this being accomplished there is nothing to do, but to multiply and divide, and the work is done.

To this end the nature of every question must be considered, and the circumstances on which the proportion depends, observed, and common sense will direct this if the terms of the question be understood.

The method of proof is by inverting the order of the question.

Note 1. If the first and third terms, both or either, be of different denominations, both terms must be reduced to the lowest denomination mentioned in either, before stating the question.

2. If the second term consists of different denominations, it must be reduced to the lowest denomination ; the fourth term, or answer will then be found in the same denomination, and must be reduced back again to the highest denomination possible.

3. After division if there be any remainder, and the quotient be not in the lowest denomination, it must be reduced to the next less denomination, dividing as before. So continue to do, till it is brought to the lowest denomination, or till nothing remains.

4. In every question there is a supposition and a demand ; the supposition is implied in the two first terms of the statement, the demand in the third.

5. When any of the terms are given in *Federal Money*, the operation is conducted in all respects as in simple numbers, observing only to place the point, or separatrix between dollars and cents, and to point off the results according to what has been taught already in *Decimal Fractions*, *Federal Money*, and further illustrated in *Compound Division*.

6. When any number of barrels, bales, or other packages, or pieces are given, if they be of equal contents, find the contents of one barrel or piece, &c. in the lowest denomination mentioned, which multiply by the number of pieces, &c. the product will be the contents of the whole. If the pieces &c. be of unequal contents find the content of each, add these together, and the sum of them will be the whole quantity.

7. The term which asks the question, or that which implies the demand is generally known by some of these words going before it; how much? How many? How long? What cost? What will? &c.

EXAMPLES.

1. If 9lbs. of tobacco cost 6*s*. what will 25lbs. cost?

OPERATION.

```
    lbs.  s.    lbs.
As  9  :  6 : : 25 : to the answer.
          25
          --
          30
         12
       ---------s.  d.
       9)150( 16  8 answer.
         9
        --
         60
         54
        --
          6
         12
        ---
       9)72(8
         72
        ---
         00
```

Here 25lbs. which asks the question, (*what will 25lbs. &c.*) is made the third term, by being put in the third place; 9lbs. being of the same name, the first term, and 6*s*. of the same name with the term sought, the second term.

I Multiply the second and third terms together and divide by the first. The remainder (6) I reduce to pence, and divide as before. The quotients make the answer, 16*s* 8.

By inverting the order of the question it will stand thus,

2. If 6*s*. buy 9lbs. of tobacco, what will 16*s* 8 buy?

```
 s.       s.  d.
 6        16  8
12        12
---      ---
72 pence 200 pence.

     pence. lbs. pence.
  As, 72  :  9 : : 200
                   200
                  ----
        72)1800(25 lbs. Answer.
           144
           ---
           360
           360
           ---
```

Here the term which asks the question (16*s* 8) is of different denominations; it must, therefore, be reduced to the lowest denomination mentioned (*pence*) as must also the other term of the same name, consequently, to be the first term.

Again—By inverting the order of the question.

3. If 16*s*8 (=200 pence) buy 25lbs. of tobacco, how much will 6*s*. (=72 pence) buy?

OPERATION.

```
    d.   lbs.     d.
As 200 : 25  : : 72
         72
        ----
         50
        175
       -----
2|00)18|00(9lbs. Ans.
       18
      ----
```

These three questions are only the first varied; they shew how any question in this Rule, may be inverted.

4. If 1oz. of silver cost 6*s*9, what will be the price of a silver cup that weighs 9oz. 4pwts. 16grs.?

```
oz.       s.  d.          oz.   pwts.  grs.
 1        6   9            9     4     16
20       12               20
----     ----            ----
20pwt.   81pence         184pwt.
24                        24
----                     ----
80                       752
40                       368
----                     ----
480grs.                  4432grs.
```

As each of the terms contains different denominations, they must all be reduced to the lowest denomination mentioned.

```
   grs.    d.      grs.
As 480  :  81  : : 4432
          4432
          ----
           162
          243
         324
        324
       -------  d.  q.
480)358992(747  3 3/5  Answer, which must be reduced to the highest denomination; thus,
    3360
    ----                 12)747  3 3/5 q.
     2299                -----------
     1920                2|0)6|2  3d.
     ----                    £3  2s. 3d. 3 3/5 q. Ans.
      3792
      3360
      ----
       432
         4
      ----
      )1728(3
       1440
       ----
        288
```

5. If 6 horſes eat 21 buſhels of oats in 3 weeks, how many buſhels will 20 horſes eat in the ſame time?

Ans. 70 *buſhels.*

The ſame queſtion inverted.

6. If 20 horſes eat 70 buſhels of oats in 3 weeks, how many buſhels will 6 horſes eat in the ſame time?

Ans. 21 *buſhels.*

The ſtatement of every queſtion requires thought and conſideration;—there are *four* numbers given in the queſtion; to know which three are to be employed in the ſtatement there can be no difficulty if the Scholar proceed deliberately and as his rule directs—firſt, conſider which of the given numbers it is, that aſks the queſtion; that determined on, put it in the third place, then ſeek for another number of the ſame name, or kind, put that in the firſt place, the ſecond place muſt now be occupied by that number which is of the ſame name or kind with the number ſought; when theſe ſteps are cautiouſly followed, the Scholar cannot fail to make his ſtatement right.

7. If an Ingot of ſilver weigh 36oz. 10pwt. what is it worth at 5s. per ounce? *Ans.* £9 2*s.* 6*d.*

8. A Goldſmith ſold a tankard for £10 12s. at the rate of 5s. 4d. per ounce, I demand the weight of it. *Ans.* 39*oz.* 15*pwt.*

9. If a family of 10 persons spend 3 bushels of malt in a month, how many bushels will serve them when there are 30 in the family?

Ans. 9 *bushels.*

10. If a family of 30 persons spend 9 bushels of malt in a month, how many bushels will serve a family of 10 persons, the same time?

Ans. 3 *bushels.*

11. If 12 acres, 3 roods produce 78 quarters, 3 pecks, how much will 35 acres, 1 rood, 20 poles, produce? *Ans.* 216 *quarters*, 5 *bushels*, 1½ *peck.*

12. If 5 acres 1 rood, produce 26 quarters 2 bushels, how many acres will be required to produce 47 quarters 4 bushels? *Ans.* 9 *acres*, 2 *roods*.

13. If 365 men consume 75 barrels of provision in 9 months, how much will 500 men consume in the same time? *Ans.* $102\frac{54}{73}$ *barrels*.

14. If 500 men consume $102\frac{54}{73}$ barrels of provision, in 9 months, how much will 365 men consume in the same time?

OPERATION.

barrels.
$102\frac{54}{73}$
Multiply by 73 *the denominator of the fraction.*
306
714
Add 54 *the numerator.*

As 500 : 7500 : : 365
7500

182500
2555

5|00)27375|00

73)5475(75 *Ans.*
511

365
365

NOTE. In the 14th example, in order to embrace the fraction ($\frac{54}{73}$ *of a barrel)* the integers, 102 barrels must be multiplied by the denominator of the fraction, (73) and the numerator, (54) added to the product.

After division, the quotient must be divided by the denominator of the fraction, and this last quotient will be the answer, all which may be seen in the example.

The Scholar must remember to do the same in all similar cases.

15. If I give 6 Dolls. for the use of 100 Dolls. for 12 months, what must I give for Dolls. 357,82 the same length of time?

OPERATION.

```
   D.   D.     D. Cts.
As 100 : 6 : : 357,82
                    6
   ---------------------D.cts.m.
        100)2146,92(21,469⅕ Ans.
            200
            ---
             146
             100
             ---
              469
              400
              ---
               692
               600
               ---
                920
                900
                ---
                 20
```

Here in the third term I had two decimal places, (82) or places of cents multiplied by the second term (6) I point off two places for cents (,92) in the product, which divided by 100, I point off three decimal places in the quotient equal to the number of decimal places in the dividend (,92 *cents and* 0 *annexed to the remainder)* there being no decimals in the divisor.

16. How much land at \$2,50 per acre should be given in exchange for 360 acres, at \$3,75 per acre? *Ans.* 540 *acres.*

The Scholar is desired to invert and prove the question.

17. If I buy 7lb. of Sugar for 75 cents, how much can I buy for 6 dollars? *Ans.* 56*lb.*

N. B. Sums in Federal Money are of the same denomination when the decimal places in each are equal.

To reduce sums in federal money to the same denomination, annex so many cyphers to that sum which has the least number of decimal places, or places of cents, mills, &c. as shall make up the deficiency.

18. If I buy 76 yards of cloth for \$113,17 what did it cost per Ell English? *Ans.* \$1,861.

19. A man spends \$3,25 per week, what is that per annum? *Ans.* \$169,464.

20. Bought a silver cup weighing 9oz. 4pwt. 16grs. for £3 2s. 3d. $3\frac{1}{5}$q. what was that per ounce? *Ans.* 6s. 9d.

21. There is a ciftern, which has 4 cocks; the firft will empty in 10 minutes; the fecond, in 20 minutes; the third in 40 minutes; and the fourth in 80 minutes; in what time will all four running together empty it?

Min.		Cift.		Min.		Cift.
10						6
20	:	1	: :	60	:	3
40						1,5
80						,75

In 1 hour the 4 cocks would empty 11,25 *Cift.*

Then,

Cift. *Min.* *Cift.* *Min.*
As 11,25 : 60 : : 1 : 5,33 *Ans.*

22. A man having a piece of land to plant, hired two men and a boy to plant it, one of the men could plant it in 12 days, the other in 15 days, and the boy in 27 days; in how long time would they plant it if they all worked together?

Ans. 5,346 *days.*

23. A Merchant bought 270 quintals of cod fifh, for $780; freight $37,70; duties and other charges $30,60; what muft he fell it at, per quintal to gain $143 in the whole? *Ans.* $3,671.

The fum of all the expenfes of the fifh with the Merchant's gain muft be found for the fecond term.

24. If a ftaff 5ft. 8in. in length caft a fhadow of 6 feet; how high is that fteeple whofe fhadow meafures 153 feet?

Ans. 144½ *feet.*

25. Bought 12 pieces of cloth each 10 yards at $1,75 per yard, what came they to? *Ans.* $210.

26. Bought 4 pieces of Holland, each containing 24 Ells-English, for $96; how much was that per yard? *Ans.* 80 *Cents.*

27. Bought 9 Chests of tea, each weighing 3C. 2qrs. 21 lb. at £4 9s. per cwt. what came they to? *Ans.* £147 13s. 8¼d.

28. A Bankrupt owes in all 972 dollars, and his money and effects are but \$607,50; what will a creditor receive on \$11,333? *Ans.* \$7,083.

29. A owes B £3475, but B compounds with him for 13s. 4d. on the pound; what must he receive for his debt? *Ans.* £2316 13*s*. 4*d*.

30. If a person whose rent is \$145 pays \$12,63 of parish taxes, how much should a person pay whose rent is \$378. *Ans.* \$32,925.

INVERSE PROPORTION.

IN ſome queſtions the number ſought becomes leſs, when the circumſtances from which it is derived become greater. Thus, when the price of goods increaſes the quantity which may be bought for a given ſum, is ſmaller. When the number of men employed at work is increaſed, the time in which they may complete it becomes ſhorter; and, when the activity of any cauſe is increaſed, the quantity neceſſary to produce any given effect is diminiſhed.

Theſe and the like caſes belong to the

SINGLE RULE OF THREE INVERSE.

The Single Rule of Three Inverſe teaches, by having three numbers given to find a fourth, having the ſame proportion to the ſecond, as the firſt has to the third.

If more require leſs, or leſs require more, the queſtion belongs to the Single Rule of Three Inverſe.

More requiring leſs, is when the third term is greater than the firſt, and requires the fourth term to be leſs than the ſecond.

Leſs requiring more, is when the third term is leſs than the firſt, and requires the fourth term to be greater than the ſecond.

RULE.

"State and reduce the terms as in the rule of three direct; then multiply the firſt and ſecond terms together, divide the product by the third, and the quotient will be the anſwer in the ſame denomination with the ſecond term."

EXAMPLES.

1. If 48 men can build a wall in 24 days, how many men can do the ſame in 192 days?

OPERATION.

```
   Men.   Days.    Men.
As 48  :  24  : :  192
          48
         ----
          192
          96
       -------
192)1152(6 Ans.
    1152
```

Here the third term is greater than the firſt, and common ſenſe teaches the fourth term, or anſwer muſt be *leſs* than the ſecond, for if 48 men can do the work in 24 days, certainly 192 men will do it in leſs time. In this way it may be determined if a queſtion belong to the Rule of Three Inverſe.

2. If a board be 9 inches broad, how much in length will make a ſquare foot?

```
   InB.  InL.   InB.  InL.
As 12  :  12  : :  9  :  16 Ans.
```

3. How many yards of ſarcenet, 3qrs. wide, will line 9 yards of cloth of 8qrs. wide? *Ans.* 24 *yards*.

4. Lent a friend 292 dollars for 6 months; some time afterwards, he lent me 806 dollars; how long may I keep it to balance the favor?

Ans. 2 *months* 5 *days.*

5. A garrison had provisions for 8 months at the rate of 15 ounces to each person per day; how much must be allowed per day in order that the provisions may last $9\frac{1}{2}$ months?

Ans. $12\frac{12}{19}$ *ounces.*

6. A garrison of 1200 has provisions for 9 months at the rate of 14 ounces per day, how long will the provisions last at the same allowance if the garrison be reinforced by 400 men?

Ans. $6\frac{3}{4}$ *months.*

7.——How must the daily allowance be in order that the provisions may last 9 months after the garrison is reinforced?

Ans. $10\frac{1}{2}$ *ounces.*

8. If a man perform a journey in 15 days, when the day is 12 hours long, in how many will he do it when the day is but 10 hours? *Ans.* 18 *days.*

9. If a piece of land 40 rods in length, and 4 in breadth make an acre, how wide must it be, when it is but 25 rods long? *Ans.* $6\frac{2}{5}$ *rods.*

10. There was a certain building raised in 8 months by 120 workmen, but the same being demolished it is required to be rebuilt in 2 months; I demand how many men must be employed about it? *Ans.* 480 *men.*

11. How much in length, that is 3 inches broad, will make a square foot? *Ans.* 48 *inches.*

12. There is a cistern, having 1 pipe which will empty it in 10 hours; how many pipes of the same capacity will empty it in 24 minutes? *Ans.* 25 *pipes.*

13. If a field will feed 6 cows 91 days, how long will it feed 21 cows? *Ans.* 26 *days.*

GENERAL RULE

For stating all questions whether direct or inverse.

1. PLACE that number for the third term, which signifies the same kind of thing, with what is sought, and consider whether the number sought will be greater or less. If greater place the least of the other terms for the first; but if less, place the greater for the first, and the remaining one for the second term.

2. Multiply the second and third terms together, divide the product by the first, and the quotient will be the answer.

EXAMPLES.

1. If 30 horses plough 12 acres, how many will forty plough in the same time?

OPERATION.

```
H.   H.      Ac.
30 : 40 : : 12
     12
    ----
30)480(16 Ans.
```

Here because the thing sought is a number of acres, we place 12, the given number of acres for the third term; and because 40 horses will plough more than 12, we make the lesser number, 30, the first term and the greater number, 40 the second term.

2. If 40 horses be maintained for a certain sum on hay at 5 cents per stone, how many will be maintained, on the same sum, when the price of hay rises to 8 cents per stone?

```
C.  C.    H.
8 : 5 : : 40
    40
   ----
 8)200(25 Ans.
   16
   ---
    40
    40
```

Here, because a number of horses is sought we make the given number of horses, 40, the third term, and because fewer will be maintained for the same money, when the price of hay is dearer, we make the greater price, 8 cents, the first term, and the lesser price, 5 cents the second.

The first of these examples is *direct*, the second *inverse*.

Every question consists of a supposition and a demand.

In the first the supposition is, that 30 *horses plough* 12 *acres*, and the demand *how many* 40 *horses will plough?* And the first term of the proportion, 30, is found in the supposition, in this and every other *direct question*.

In the second, the supposition is that 40 *horses are maintained on hay at* 5 *cents per stone*, and the demand, *how many will be maintained on hay at* 8 *cents?* And the first term of the proportion, 8, is found in the demand, in this and every other *inverse question*.

3. If a quarter of wheat afford 60 ten penny loaves, how many eight penny loaves may be obtained from it? *Ans.* 75 *loaves.*

4. If in 12 months, 100 dollars gain 6 dollars interest, what will gain the same sum in 5 months? *Ans.* 240 *dollars.*

Supplement to the Single Rule of Three.

QUESTIONS.

1. What is the Single Rule of Three; or, the Rule of Proportion?
2. How many kinds of Proportion are there?
3. What is it that the Single Rule of Three Direct teaches?
4. How can it be known that a queſtion belongs to the Single Rule of Three Direct?
5. What is underſtood by *more requiring more*, and *leſs requiring leſs?*
6. How are queſtions in the Rule of Three ſtated?
7. Having ſtated the queſtion, how is the anſwer found in direct Proportion?
8. What do you obſerve of the firſt and third terms concerning the different denominations, ſometimes contained in them?
9. When the ſecond term contains different denominations, what is to be done?
10. How is it known what denomination the quotient is of?
11. If the quotient, or anſwer, be found in an inferior denomination, what is to be done?
12. When the terms are given in Federal Money, how is the operation conducted?
13. How are ſums in Federal Money reduced to the ſame denomination?
14. When any number of barrels, bales, or pieces, &c. are given, what is the method of procedure?
15. What is it that the Single Rule of Three in Inverſe teaches?
16. How are queſtions ſtated in Inverſe Proportion?
17. What is underſtood by *more requiring leſs*, and *leſs requiring more?*
18. How is the anſwer found in the Rule of Three Inverſe?
19. What is the general Rule for ſtating all queſtions, whether Direct or Inverſe?

EXERCISES.

1. If my horſe and ſaddle are worth 18 guineas, and my horſe be worth ſix times as much as my ſaddle, pray what is the value of my horſe?

Ans. $72.

2. How many yards of mattin, that is, half a yard wide, will cover a room that is 18 feet wide, and 30 feet long?

Ans. 120 *yards*.

3. Suppose 800 soldiers were placed in a garrison, and their provisions were computed sufficient for two months; how many soldiers must depart that the provisions may serve them 5 months?

Ans. 480.

4. I borrowed 185 quarters of corn when the price was 19*s.* how much must I repay, to indemnify the lender, when the price is 17*s.* 4*d.*

Ans. $202\frac{1}{52}$.

5. A and B depart from the ſame place and travel the ſame road; but A goes 5 day before B at the rate of 20 miles per day; B follows at the rate of 25 miles per day; in what time and diſtance will he overtake A?

Ans. B will overtake A in 20 days and travel 500 miles.

Here two ſtatements will be neceſſary; one to aſcertain the time, and another to aſcertain the diſtance.

METHOD

Of aſſeſſing town or pariſh taxes.

1. An inventory of the value of all the eſtates, both real and perſonal, and the number of polls, for which each perſon is ratable, muſt be taken in ſeparate columns. Then to know what muſt be paid on the dollar, make the total value of the inventory the firſt term; the tax to be aſſeſſed, the ſecond, and 1 dollar, the third, and the quotient will ſhew the value on the dollar.

NOTE. This method is taken from Mr. PIKE's Arithmetic, with this difference, that here the money is reduced to Federal Currency.

2. Make a table, by multiplying the value on the dollar by 1,2,3,4,5, &c.
3. From the Inventory take the real and perſonal eſtates of each man, and find them ſeparately, in the table, which will ſhew you each man's proportional ſhare of the tax for real and perſonal eſtates.

If any part of the tax be averaged on the polls, before ſtating to find the value on the dollar, deduct the ſum of the average tax from the whole ſum to be aſſeſſed; for which average make a ſeparate column as well as for the real and perſonal eſtates.

EXAMPLES.

Suppoſe the General Court ſhould grant a tax of 150,000 dollars, of which a certain town is to pay Dolls. 3250,72 and of which the polls being 624 are to pay 75 cents, each; the town's inventory is 69568 dollars; what will it be on the dollar; and what is A's tax (as by the inventory) whoſe eſtate is as follows, viz. real 856 dollars; perſonal 103 dollars; and he has 4 polls?

Pol. *Cts.* *Pol.* *Dolls.*
1. As 1 ; ,75 : : 624 : 468 the average part of the tax to be deducted from $3250,72 and there will remain $2782,72.

Dolls. *Dolls. Cts.* *Doll. Cts.*
2. As 69568 : 2782, 72 : : 1 : 4 on the dollar.

TABLE.

Dolls.	*Dolls. cts.*	*Dolls.*	*Dolls. cts.*	*Dolls*	*Dolls.*
1 is	4	20 is	80	200 is	8
2 —	8	30 —	1 20	300 —	12
3 —	12	40 —	1 60	400 —	16
4 —	16	50 —	2 00	500 —	20
5 —	20	60 —	2 40	600 —	24
6 —	24	70 —	2 80	700 —	28
7 —	28	80 —	3 20	800 —	32
8 —	32	90 —	3 60	900 —	36
9 —	36	100 —	4 00	1000 —	40
10 —	40				

Now to find what A's rate will be.

His real eſtate being 856 dollars I find by the Table that 800 dollars is $32 cts.
that 50 — — 2
that 6 — — 0 24

Therefore the tax for his real eſtate is 34 24
In like manner I find the tax for his perſonal eſtate to be } 4 12
His 4 polls, at 75 cents each, are 3

41 36

Real. *Dolls. Cts.*	Perſonal. *Dolls. Cts.*	Polls. *Dolls. Cts.*	Total. *Dolls. Cents.*
34 24	4 12	3 0	41 36

§ 8. Double Rule of Three.

THE Double Rule of Three, ſometimes called COMPOUND PROPORTION, teaches, by having five numbers given to find a ſixth, which, if the proportion be *direct*, muſt bear the ſame proportion to the fourth and fifth, as the third does to the firſt and ſecond. But if the proportion be *inverſe*, the ſixth number muſt bear the ſame proportion to the fourth and fifth, as the firſt does to the ſecond and third.

RULE.

"1. State the queſtion, by placing the three conditional terms in ſuch order, that that number which is the cauſe of gain, loſs, or action, may poſſeſs the firſt place; that which denotes ſpace of time, or diſtance of place, the ſecond; and that which is the gain, loſs, or action, the third."

"2 Place the other two terms, which move the queſtion, under thoſe of the ſame name."

"3. Then, if the blank place, or term ſought, fall under the third place, the proportion is direct, therefore multiply the three laſt terms together, for a dividend, and the other two for a diviſor; then the quotient will be the anſwer."

"4 But if the blank fall under the firſt or ſecond place, the proportion is inverſe, wherefore, multiply the firſt, ſecond, and laſt terms together, for a dividend, and the other two, for a diviſor; the quotient will be the anſwer."

EXAMPLES.

1. If 100 dollars gain 6 dollars, in 12 months, what will 400 dollars gain in 8 months?

Statement of the queſtion.

D.		M.		D.	
100	:	12	: :	6	*Terms in the ſuppoſition, or conditional terms.*
400	:	8			*Terms which move the queſtion.*

Of the three conditional terms, it is evident, that 100 dollars put at intereſt is that one, which is the cauſe of gain; conſequently, 100 dollars muſt be the firſt term; and becauſe, 12 months is the ſpace of time in which the gain is made, this muſt be the ſecond term; and 6 dollars which is the gain, the third term. The other two terms muſt then be arranged under thoſe of the ſame name.

Now as the blank falls under the third place, therefore, the queſtion is in direct proportion, and the anſwer is found by multiplying the three laſt terms together for a dividend and the two firſt for a diviſor.

OPERATION.

```
          100 : 12 : : 6
          400 :  8                Then, 12|00)192|00(
            8                         ---------
          ----                         Dolls. 16 Anſwer.
  100     3200
   12        6
  ----    -----
 1200 Div. 19200 Dividend.
```

2. If 100 dollars gain 6 dollars in 12 months, in what time will 400 dollars gain 16?

OPERATION.

D.	M.	D.
100 :	12 : :	6
400		16
6		12
2400 *divis.*		192
		100
		19200 *divided.*

Here the blank falling under the second term, the proportion is indirect.

Therefore multiply the first, second and last terms together for a dividend, and the other two for a divisor.

Then, 24|00) 192|00(8 *Ans.* M.
192

3. A farmer sells 204 dollars worth of grain, in 5 years, when it is sold at 60 cents per bushel; what is it per bushel when he sells 1000 dollars worth, in 18 years, if he sell the same quantity yearly?

Cts. *Y.* *D.*
60 : 5 : : 204 *cts.m.*
18 : : 1000 : ,816 *Ans.*

4. If 7 men can reap 84 acres of wheat in 12 days; how many men can reap 100 acres in 5 days?

M. *D.* *A.*
7 : 12 : : 84 *M.*
5 : : 100 : 20 *Ans.*

Supplement to the Double Rule of Three.

QUESTIONS.

1. WHAT is the Double Rule of Three; or COMPOUND PROPORTION?
2. How are queſtions to be ſtated in the Double Rule of Three?
3. How is it known, after the ſtatement of the queſtion, whether the proportion be direct or inverſe?
4. When the proportion is Direct, how is the anſwer to be found?
5. When the proportion is Inverſe, how is the anſwer to be found?

EXERCISES.

1. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet wide in 16 days, in what time will 24 men build one 200 feet long, 8 feet high and 6 thick? *Ans.* 80 *days.*

The ſolid contents in each piece of wall, according to the given dimenſions, muſt be found before ſtating the queſtion.

2. If the freight of 12Cwt. 2qrs. 6lb. 275 miles, cost $27,78; how far may 60cwt. 3qrs. be shipped for $234,78? *Ans.* 480 *miles.*

3. An usurer put out 75 dollars, at interest; and at the end of 8 months received for principal and interest, 79 dollars; I demand at what rate per cent he received interest? *Ans.* 8 *per cent.*

4. If 7 men can make 84 rods of wall in 6 days; in what time will 10 men make 150 rods? *Ans.* $7\frac{1}{2}$ *days.*

5. If the freight of 9hhd. of ſugar, each weighing 12 Cwt. 20 leagues, coſt £16 : what muſt be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$Cwt. 100 leagues ? *Ans.* £92 11 $10\frac{2}{9}$.

§ 9. Practice.

" PRACTICE is a contraction of the Rule of Three Direct, when the first term happens to be an unit, or one; it has its name from its daily use among Merchants and Tradesmen, being an easy and concise method of working most questions, which occur in trade and business.

PROOF. By the Single Rule of Three, Compound Multiplication, or by varying the parts.

Before any advances are made in this rule, the Learner must commit to memory, the following

TABLES.

Aliquot, or even parts of Money.

Pts. of a shill. of a £.		
d.	*s.* and	*£.*
6	is $\frac{1}{2}$ —	$\frac{1}{40}$
4	— $\frac{1}{3}$ —	$\frac{1}{60}$
3	— $\frac{1}{4}$ —	$\frac{1}{80}$
2	— $\frac{1}{6}$ —	$\frac{1}{120}$
$1\frac{1}{2}$	— $\frac{1}{8}$ —	$\frac{1}{160}$
1	— $\frac{1}{12}$ —	$\frac{1}{240}$
$\frac{3}{4}$	— $\frac{1}{16}$ —	$\frac{1}{320}$
$\frac{1}{2}$	— $\frac{1}{24}$ —	$\frac{1}{480}$
$\frac{1}{4}$	— $\frac{1}{48}$ —	$\frac{1}{960}$

5*d.* is the sum of 4*d.* and 1*d.*
7*d.* —— 6*d.* and 1*d.*
8*d.* is twice 4*d.*
9*d.* is the sum of 6*d.* and 3*d.*
10*d.* —— 6*d.* and 4*d.*
11*d.* —— 6*d.* 3*d.* & 2*d.*

Pts. of a pound.		
s.	*d.*	is £.
10	0 —	$\frac{1}{2}$
6	8 —	$\frac{1}{3}$
5	0 —	$\frac{1}{4}$
4	0 —	$\frac{1}{5}$
3	4 —	$\frac{1}{6}$
2	6 —	$\frac{1}{8}$
1	8 —	$\frac{1}{12}$
1	4 —	$\frac{1}{15}$
1	3 —	$\frac{1}{16}$
1	0 —	$\frac{1}{20}$
0	10 —	$\frac{1}{24}$
0	8 —	$\frac{1}{30}$
0	5 —	$\frac{1}{48}$
0	$2\frac{1}{2}$ —	$\frac{1}{96}$

Practice admits of a great variety of cases, the multiplicity of which serves little else, than that of confounding the mind of the Scholar; a different method will be pursued here and the whole comprised, in a few cases, such as shall be useful and easy for the Scholar to bear in his memory.

The small number of examples under each case will be made up in the supplement; this will lead the Scholar to a more particular consideration of them.

OPERATIONS.

Pounds, Shill. Pence, Farthings.

When the price of the given quantity is 1£. 1*s.* 1*d.* per pound, yard, &c. then will the quantity itself be the answer at the supposed price. Therefore,

CASE 1.

When the price of 1*yd.* 1*lb* &c. *consists of farthings only;* if it be one farthing, take a fourth of the quantity; if a half penny, take a half; if three farthings take a half and a fourth of the quantity and add them. This gives the value in pence, which must be reduced to pounds.

Dollars, Cents, Mills.

RULE.

Multiply the quantity by the price of 1 pound, yard, &c. the product will be the answer.

Pounds, Shill. Pence, Farthings.

EXAMPLES.

1. What will 362 yards cost, at $\frac{1}{2}d$. per yard?

OPERATION.

2)362
12)181 *pence.*

15*s.* 1*d.* *Ans.*

Here the quantity stands for the price at one penny per yard, but as two farthings, are but half one penny, therefore dividing the quantity by 2 gives the price at half a penny per yard, which must be reduced to shillings.

2. What will $354\frac{1}{2}$ yards cost, at $\frac{1}{4}d$ per yard?

OPERATION.

d.	*q.*
4)354	2
12)88	2
7*s.*4*d.*	2 *Ans.*

3. What will 263 yards cost at 3q. per yard? *Ans.* 16*s.* $5\frac{1}{4}d$.

4. What will 816 yards cost at 1q. per yard? *Ans.* 17*s.*

Dollars, Cents, Mills.

1. What will 362 yards cost at 7 mills per yard?

OPERATION.

3 6 2 *quantity.*
,0 0 7 *price.*

$2, 5 3 4 *Answer.*

NOTE. The answers in the different kinds of money will not always compare, because in the reduction of the price, a small fraction is often lost or gained.

2. What will $354\frac{1}{2}$ yards cost, at 3 mills per yard?

OPERATION.

3 5 4 ,5 *quantity.*
,0 0 3 *price.*

$10 ,6 3 5 *Answer.*

3. What will 263 yards cost, at 1 cent per yard? *Ans.* $2,63.

4. What will 816 yards cost at 3 mills per yard? *Ans.* $2,448.

Pounds, Shill. Pence, Farthings.

5. What will 97 yards cost at 3q. per yard? *Ans.* 6*s.* 0$\frac{3}{4}$*d.*

6. What will 126 yards cost at $\frac{1}{2}$*d.* per yard? *Ans.* 5*s.* 3*d.*

CASE 2.

When the price of 1*lb.* 1 *yard, &c. consists of pence, or of pence and farthings;* if it be an even part of a shilling, find the value of the given quantity at 1s. per yard, *(the quantity itself expresses the price at* 1*s. per yard; if there are quarters, &c. write for* $\frac{1}{4}$ 3*d. for* $\frac{1}{2}$ 6*d. for* $\frac{3}{4}$ 9*d.)* and divide by that *even part,* which the price is of 1 shilling. If the price be not an aliquot or even part of 1 shilling, it must be divided into two or more aliquot parts; calculate for these separately, and add the values; the answer will be obtained in shillings, which must be reduced to pounds.

Dollars, Cents, Mills.

5. What will 97 yards cost, at 1 cent per yard? *Ans.* ,97 *cts.*

6. What will 126 yards cost at 7 mills per yard? *Ans.* $0,882.

Pounds, Shill. Pence, Farthings.

EXAMPLES.

1. What will 476 yards cost, at $7\frac{1}{2}d.$ per yard?

OPERATION.

		s.
6*d.*	$\frac{1}{2}$	476 *Price at* 1*s. per yard.*
$1\frac{1}{2}$*d.*	$\frac{1}{4}$	238 *Price at* 6*d. per yard.*
		59 6*d. price at* $1\frac{1}{2}$*d. per yd.*
		20)29,7 6*d. price at* $7\frac{1}{2}$*d. per yd.*
		£14 17*s.* 6*d. Answer.*

PROOF.

1. *By the Rule of Three,*

	Y.	*£.*	*s.*	*d*		*Y.*
As	476	: 14	17	6	: :	1
		20				
		297				
		12				
	476)3570(7*d.*					
	3332					
	238					
	4					
	)952(2q.					
	952					

2. *By Compound Multiplication.*

£.	*s.*	*d.*
		$7\frac{1}{2}$ *price of* 1 *yard.*
		10
	6	3 *price of* 10 *yards.*
		10
3	2	6 *price of* 100 *yards.*
		4
12	10	0 *price of* 400 *yards.*
2	3	9 *price of* 70 *yards.*
	3	9 *price of* 6 *yards.*
£14	17	6 *price of* 476 *yards.*

Dollars, Cents, Mills.

7. What will 476 yards come to at 10 cents 4 mills per yard?

OPERATION.

476
,104
1904
4760
$49,504 *Ans.*

PROOF.

cts.	*m.*	*D.*	*cts.*	*m.*	*yds.*
,1 0	4)4	9	5 0	4(	476
	4	1	6		
		7	9 0		
		7	2 8		
			6 2	4	
			6 2	4	

Pounds, Shill. Pence, Farthings.

2. What will 176 yards cost, at $9\frac{1}{2}d.$ per yard?

OPERATION.

		s.
6d.	$\frac{1}{2}$	176 *value at* 1s. *per yd.*
3d.	$\frac{1}{2}$	88 *value at* 6d. *per yd.*
$\frac{1}{2}d.$	$\frac{1}{6}$ of	44 *value at* 3d. *per yd.*
		7 4d. *value at* $\frac{1}{2}d.$ *per yd.*
		2(0)13(9 4d.—*at* $9\frac{1}{2}d.$ *per yd.*
		£6 19s. 4d. *Ans.*

PROOF.

3. What will $568\frac{1}{4}$ yards cost at 7d. per yard?

Ans. £16 11s. $5\frac{3}{4}d.$

Dollars, Cents, Mills.

8. What will 176 yards cost, at 13 cents, 2 mills per yard?

Ans. $23,232.

9. What will $568\frac{1}{4}$ yards cost at 9 cents, 7 mills per yard?

Ans. $55,12.

Pounds, Shill. Pence, Farthings.

4. What will $685\frac{3}{4}$ yards come to at $2\frac{1}{2}d.$ per yard?
Ans. £7 2*s.* $10\frac{1}{4}d.$

5. What will $649\frac{1}{4}$ yards cost, at 10d. per yard? *Ans.* £27 1*s.* $0\frac{1}{2}d.$

6. What will $683\frac{1}{4}$ yards cost at $8\frac{1}{4}d.$ per yard?
Ans. £23 10*s.* $0\frac{1}{4}d.$

Dollars, Cents, Mills.

10. What will $685\frac{3}{4}$ yards come to, at 3 cents, 5 mills per yard?
Ans. $24,001.

11. What will $649\frac{1}{4}$ yards cost, at 13 cents, 9 mills per yard?
Ans. $90,245.

12. What will $683\frac{1}{4}$ yards cost, at

Pounds, Shill. Pence, Farthings.

CASE 3.

If the price of 1lb. 1yd. &c. be shillings and pence, and an even part of 1£. Divide the value of the given quantity at 1£. per yard by that *even part*, which the price is of 1£. The quotient will be the anſwer.

EXAMPLES.

1. What will 719½ yards coſt at 1*s.* 4*d.* per yard ?

OPERATION.

		£.	s.	
1s4	$\frac{1}{15}$	719	10	*price at £1 per yd.*
		143	18	*price at 4s. per yd.*
		Ans. 47	19 4*d.*	at 1s4 *per yd.*

Here for the ſake of eaſe in the operation, becauſe $5\times3=15$, therefore I divide the price at one pound *per yard* by 5, and that quotient by 3, which gives the anſwer.

2. What will 648 yards coſt, at 1*s.* 8*d.* per yard ? *Ans.* £54.

Dollars, Cents, Mills.

13. What will 719½ yards coſt, at 22 cents, 3 mills per yard ?
Ans. $160,448.

14. What will 648 yards coſt, at 27 cents, 8 mills per yard ?
Ans. $180,144.

Pounds, Shill. Pence, Farthings.

3. What will 687¼ yards cost, at 5s. per yard? *Ans.* £171·17s. 6d.

CASE 4.

When the price of 1 yard, &c. is shillings, or shillings, pence & farthings and not an even part of £1. Multiply the value of the quantity at 1s. per yard by the number of shillings; for the pence and farthings take parts, as in CASE 2, the results added will give the answer, which must be reduced to pounds.

If the price be shillings only, and an even number; multiply by half the price or even number of shillings for one yard, double the unit figure of the product for shillings, the remaining figures will be pounds.

NOTE. When the quantity contains a fraction, work for the integers, and for the fraction take proportional parts of the rate.

EXAMPLES.

1. What will 167½ yards cost, at 17s. 6d. per yard?

OPERATION.

```
             s.
| 6d. | ½ |  167
              17
            ----
            1169
            167
            ----
            2839 price at 17s. per yd.
              83 6—at 6d. per yd.
               8 9 price of ½ yd.
            ----
          2|0)293|1 3d.
          --------
    Ans. £146 11s. 3d.
```

Dollars, Cents, Mills.

15. What will 687¼ yards cost, at 83 cents, 3 mills per yard? *Ans.* $572,687½

16. What will 167½ yards cost, at $2,916? *Ans.* $488,43.

Pounds, Shill. Pence, Farthings.

2. What will 5482 yards cost, at 12*s*. $4\frac{1}{2}$*d*. per yard ?
Ans. £3391 19*s*. 9*d*.

3. What will 614 yards cost, at 16*s*. per yard ?

OPERATION.

614
8 *half the price.*

4912 *double the first figure for (shillings.*
£491 4*s*. *Ans*.

4. What will 176 yards cost, at 12*s*. per yard ? *Ans*. £105 12*s*.

5. What will 36 yards cost, at 7*s*. 6*d*. per yard ? *Ans*. £13 10*s*.

Dollars, Cents, Mills.

17. What will 5482 yards cost, at $2,063 per yard ?
Ans. $11309,366.

18. What will 614 yards cost, at $2,667 per yard ?
Ans. $1637,538.

19. What will 176 yards cost, at 2 dollars per yard ? *Ans*. $352.

20. What will 36 yards cost, at $1,25 per yard ? *Ans*. $45.

Pounds, Shill. Pence, Farthings.

CASE 5.

When the price of 1yd. 1lb. &c. is pounds, ſhillings and pence; Multiply the quantity by the pounds and if the ſhillings and pence be an even part of a pound, divide the given quantity by that *even part*, and add the quotient to the product for the anſwer; but if they are not an even part of £1, take parts of parts and add them together. Or, you may reduce the pound in the price of 1 yard, &c. to ſhillings and proceed as in the caſe before.

EXAMPLES.

1. What will 59 yards coſt, at £6 7*s*. 6*d*. per yard?

OPERATION.

	£.			
5*s*. is $\frac{1}{4}$ of £1	59	*value at £1 per yd.*		
	6			
	354	—*at £6 per yard.*		
2*s*6 is $\frac{1}{2}$ of 5.	14	15*s*. *at 5s. per yd.*		
	7	7	6*d*	*at 2s6 per yd.*

Ans. £376 2 6 *at £6 7s. 9d.*

2. What will 163 yards coſt, at £2 8*s*. per yard? *Ans.* £391 4*s*.

Dollars, Cents, Mills.

21. What will 59 yards coſt, at $21,25 per yard?

OPERATION.

D.	C.
21,25	
59	
191	25
1062	5
$1253	75 *Ans.*

22. What will 163 yards coſt, at 8 dollars per yard? *Ans.* $13,04.

Pounds, Shill. Pence, Farthings.

3. What will 76 yards cost, at £3 2s. 7d. per yard?

OPERATION.

	s.	
6*d.* is $\frac{1}{2}$ *of* 1*s.*	76	*value at* 1*s. per yd.*
	62	=*shilling in* £3 2*s.*
	152	*value at* 2*s. per yd.*
	456	—*at* 60*s. per yd.*
1*d. is* $\frac{1}{6}$ of 6*d.*	38	—*at* 6*d. per yd.*
	6 4*d.*	—*at* 1*d. per yd.*
	2|0)475|6 4*d.*	

Ans. £237 16*s.* 4*d.*

4. What is the value of 84 yards, at £2 14s. per yard?
Ans. £226 16*s.*

Dollars, Cents, Mills.

23. What will 76 yards cost, at \$10,43 per yard?
Ans. \$792,68.

24. What is the value of 84 yards at 9 dollars per yard? *Ans.* \$756.

Supplement to Practice.

QUESTIONS.

1. WHAT is Practice?
2. Why is it so called?
3. When the price of 1 yard, &c. is farthings, how is the value of any given quantity found at the same rate?
4. When the price consists of pence and farthings, and is an even part of 1s. how is the value of any given quantity found?
5. When the price is pence and farthings and not an even part of 1s. what is the method of procedure?
6. When the price consists of shillings, pence and farthings, how is the value of any given quantity found?
7. When the price contains shillings and pence and is an even part of £1 how is the operation to be conducted?
8. When the price consists of shillings only, and an even number, what is the most direct way to find the value of any given quantity?
9. When the quantity contains fractions, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, &c. how are they to be treated?
10. When the price consists of pounds, and lower denominations, how is the value of any given quantity found?
11. When the prices are given in Dollars, Cents and Mills, how is the value of any given quantity found in federal money?
12. What is the method of proof?
13. How are the operations in Federal Money proved?

EXERCISES IN PRACTICE.

In the following exercises, the attention of the Scholar must be excited first to consider to which of the preceding cases each question is to be referred. That being ascertained, he will proceed in the operation according to the instruction there given.

1. What will $745\frac{3}{4}$ yards cost, at 11d. per yard? *Ans.* £34 3*s.* $7\frac{1}{4}$*d.*

Under which of the preceding cases does this question properly belong?

What must be done with the fraction ($\frac{3}{4}$ *of a yard)* in the quantity?

2. What will 964 yards cost, at 1s. 8d. per yard? *Ans.* £80 6*s*. 8*d*.

OPERATION. PROOF.

3. What will 354$\frac{1}{4}$ yards cost, at $\frac{1}{4}$*d*. per yard? *Ans.* 7*s*. 4$\frac{1}{2}$*d*.

4. What will 316 yards cost, at $\frac{3}{4}$*d*. per yard? *Ans.* 19*s*. 9*d*.

5. What will 567$\frac{1}{2}$ yards cost, at 1$\frac{1}{2}$*d*. per yard? *Ans.* £3 10*s*. 11$\frac{1}{4}$*d*.

6. What will 913$\frac{1}{2}$ yards cost, at 6d. per yard? *Ans.* £22 16*s*. 9*d*.

7. What will 912½ yards cost, at 9d. per yard?

Ans. £34 4*s.* 4½*d.*

8. What will 76 yards cost, at 2d. per yard?

Ans. 12*s.* 8*d.*

9. What will 845 yards cost, at 8s. per yard? *Ans.* £338.

10. What will 91 yards come to, at 16s. per yard? *Ans.* £72 16*s.*

11. What will 156½ yards come to, at 6s. 4d. per yard?

Ans. £49 11*s.* 2*d.*

12. What will 96 yards cost, at 10s. 1½d. per yard?

Ans. £48 12*s.*

13. What will 67½ yards cost, at 12s. 2d. per yard? *Ans.* £41 1*s.* 3*d.*

14. What will 843 yards cost, at 6s. 8d. per yard? *Ans.* £281.

15. What will 75 yards cost, at £3 3s. 4d. per yard? *Ans.* £237 10*s.*

16. What will 59 yards come to, at £6. 7s. 6d. per yard? *Ans.* £376 2*s.* 6*d.*

17. What will 59¾ yards come to, at £3 6s. 8d. per yard? *Ans.* £199 3*s.* 4*d.*

18. What will 68 yards cost, at £4 6s. per yard? *Ans.* £292 8*s.*

N. B. The following queſtions are left without any anſwers, that the Scholar may operate and prove each queſtion.

19. What will 11 yards of flannel, at 2s. 6d. per yard, come to?

OPERATION. PROOF.

20. What will 13lb. of cotton coſt, at 3s. 4d. per lb.?

21. What will 183 yards of ribbon come to, at 8d. per yard?

THE

SCHOLAR'S ARITHMETIC.

SECTION III.

Rules occasionally useful to men in particular callings and pursuits of life.

§ 1. Involution.

INVOLUTION, or the raising of powers is the multiplying of any given number into itself continually, a certain number of times. The quantities in this way produced, are called powers of the given number. Thus,

$4\times4=16$ is the 2d. power, or square of 4. $=4^2$

$4\times4\times4=64$ is the 3d. power, or cube of 4. $=4^3$

$4\times4\times4\times4=256$ is the 4th power, or biquadrate of 4. $=4^4$

The given number, (4) is called the first power; and the small figure, which points out the order of the power, is called the *Index* or the *Exponent*.

§ 2. Evolution.

EVOLUTION, or the extraction of roots, is the operation by which we find any root of any given number.

The root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th, root, &c. accordingly as it is, when raised to the 2d, 3d, 4th, &c. power, equal to that power. Thus 4 is the square root of 16, because $4\times4=16$. 4 also is the cube root of 64, because $4\times4\times4=64$; and 3 is the square root of 9, and 12 is the square root of 144, and the cube root of 1728, because $12\times12\times12=1728$, and so on.

To every number there is a root, although there are numbers, the precise roots of which can never be obtained. But, by the help of decimals, we can approximate towards those roots, to any necessary degree of exactness. Such roots are called *Surd Roots*, in distinction from those, perfectly accurate, which are called *Rational Roots*.

The square root is denoted by this character $\sqrt{}$ placed before the power; the other roots by the same character, with the index of the root placed over it. Thus the square root of 16 is expressed $\sqrt{16}$, and the cube root of 27 is $\sqrt[3]{27}$, &c.

When the power is expressed by several numbers with the sign + or — between them, a line is drawn from the top of the sign over all the parts of it; thus, the second root of 21—5 is $\sqrt{21-5}$, and the 3d. root of 56+8 is $\sqrt[3]{56+8}$, &c.

The second, third, fourth, and fifth powers of the nine digits may be seen in the following

TABLE.

Roots, -	or 1st. Powers.	1	2	3	4	5	6	7	8	9
Squares,	or 2d. Powers.	1	4	9	16	25	36	49	64	81
Cubes, -	or 3d. Powers.	1	8	27	64	125	216	343	512	729
Biquadrates,	or 4th Powers.	1	16	81	256	625	1296	2401	4096	6561
Sursolids,	or 5th. Powers.	1	32	243	1024	3125	7776	16807	32768	59049

§ 3. Extraction of the Square Root.

TO extract the square root of any number, is to find another number which multiplied by, or into itself, will produce the given number; and after the root is found, such a multiplication is a proof of the work.

RULE.

1. "Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. "Find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. "Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. "Seek how often the divisor is contained in the dividend (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor; multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. " Double the figure already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed, and continue the operation in the same manner, till you have brought down all the periods.

" NOTE 1. If, when the given power is pointed off as the power requires, the left hand period should be deficient, it must nevertheless stand as the first period.

" NOTE 2. If there be decimals in the given number, it must be pointed both ways from the place of units: If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given number are exhausted, the operation may be continued at pleasure by annexing cyphers."

EXAMPLES.

1. What is the square root of 729?

OPERATION.

```
 . .
 729(27 the root.
 4
 ----
47(329
  329
 ----
  000
PROOF.
   27
   27
 ----
  189
  54
 ----
  729
```

The given number being distinguished into periods, I seek the greatest square number in the left hand period (7) which is 4, of which the root (2) being placed to the right hand of the given number, after the manner of a quotient, and the square number (4) subtracted from the period (7) to the remainder (3) I bring down the next period (29) making for a dividend, 329. Then the double of the root (4) being placed to the left hand for a divisor, I say how often 4 in 32? *(excepting 9 the right hand figure)* the answer is 7, which I place in the root for the second figure of it, and also to the right hand of the divisor; then multiplying the divisor thus increased by the figure (7) last obtained in the root, I place the product underneath the dividend, and subtract it therefrom, and the work is done.

DEMONSTRATION.

Of the reason and nature of the various steps in the extraction of the SQUARE ROOT.

The superficial content of any thing, that is, the number of square feet, yards, or inches, &c. contained in the surface of a thing, as of a table or floor, a picture, a field, &c. is found by multiplying the length into the breadth. If the length and breadth be equal, it is a square, then the measure of one of the sides as of a room, is the root, of which the superficial content in the floor of that room, is the second power. So that having the superficial contents of the floor of a square room, if we extract the square root, we shall have the length of one side of that room. On the other hand, having the length of one side of a square room, if we multiply that number into itself, that is to raise it to the second power, we shall then have the superficial contents of the floor of that room.

The extraction of the square root, therefore has this operation on numbers, *to arrange the number of which we extract the root into a square form.* As if a man should have 625 yards of carpeting, 1 yard wide, if he extract the square

root of that number (625) he will then have the length of one ſide of a ſquare room, the floor of which, 625 yards, will be juſt ſufficient to cover.——To proceed then to the demonſtration.

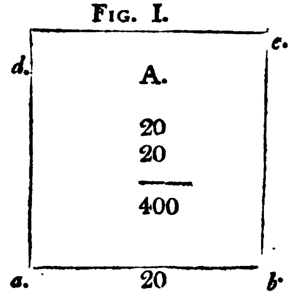
EXAMPLE 2. Suppoſing a man has 625 yards of carpeting, 1 yard wide, what will be the length of one ſide of a ſquare room, the floor of which his carpeting will cover?

The firſt ſtep is to point off the number into periods of two figures each. This determines the number of figures of which the root will conſiſt, and is done on this principle, *that the product of any two numbers can have at moſt but ſo many places of figures as there are places in both the factors, and at leaſt, but one leſs, of which any perſon may ſatisfy himſelf at pleaſure.*

OPERATION.

```
 . .
625(20
4
----
225
```

FIG. I.



The number being pointed off, as the rule directs, we find we have two periods; conſequently, the root will conſiſt of two figures. The greateſt ſquare number in the left hand period (6) is 4, of which two is the root; therefore, 2 is the firſt figure of the root, and as it is certain we have one figure more to find in the root, we may for the preſent ſupply the place of that figure by a cypher, (20) then 20 will expreſs the juſt value of that part of the root now obtained. But it muſt be remembered, that a root is the ſide of a ſquare of equal ſides. Let us then form a ſquare, A. *Fig.* I. each ſide of which ſhall be ſuppoſed 20 yards. Now the ſide *a b* of this ſquare, or either of the ſides, ſhews the root, 20, which we have obtained.

To proceed then by the rule, "*place the ſquare number underneath the period ſubtract, and to the remainder bring down the next period.*" Now the ſquare number (4) is the ſuperficial content of the ſquare A—made evident thus—each ſide of the ſquare A, meaſures 20 yards, which number multiplied into itſelf, produces 400, the ſuperficial contents of the ſquare A; alſo the ſquare number, or the ſquare of the figure 2 already found in the root, is 4, which placed under the period (6) as it falls in the place of hundreds, is in reality 400, as might be ſeen alſo by filling the places to the right hand with cyphers, then 4 ſubtracted from 6 and to the remainder (2) the next period (25) being brought down, it is plain, the ſum 225 has been diminiſhed by the deduction of 400, a number equal to the ſuperficial contents of the ſquare A.

Hence, *Fig.* I. exhibits the exact progreſs of the operation. By the operation, 400 yards of the carpeting have been diſpoſed of, and by the figure is ſeen the diſpoſition made of them.

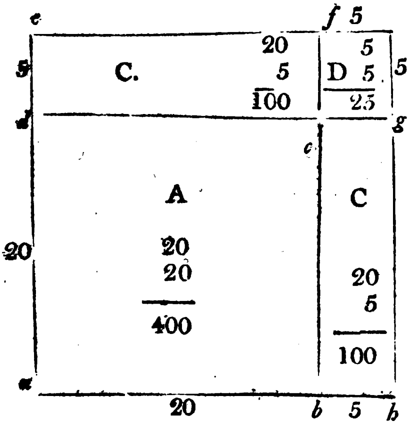
Now the ſquare A, is to be enlarged by the addition of the 225 yards which remain, and this addition muſt be ſo made that the figure, at the ſame time, ſhall continue to be a complete and perfect ſquare. If the addition be made to *one ſide* only, the figure would loſe its ſquare form; it muſt be made to *two ſides*; for this reaſon the rule directs, "*place the double of the root already found on the left hand of the dividend for a diviſor.*" The double of the root is juſt equal to two ſides *b c* and *c d* of the ſquare, A, as may be ſeen by what follows.

OPERATION *continued.*

```
  · ·
  625(25
  4
  ———
45)225
   225
   ———
   000
```

The double of the root is 4 which placed for a divisor in place of tens *(for it must be remembered, that the next figure in the root is to be placed, before it)* is in reality 40, equal to the sides *b c* (20) and *c d* (20) of the square A.

FIG. II.



Again, by the rule, "*Seek how often the divisor is contained in the dividend (except the right hand figure) and place the answer in the root, for the second figure of it, and on the right hand of the divisor.*"

Now if the sides *b c* and *c d* of the square A, *Fig.* II. is the length to which the remaining 225 yards are to be added, and the divisor *(4 tens)* is the sum of these two sides, it is then evident, that 225 divided by the length of the two sides, that is by the divisor *(4 tens)* will give the breadth of this new addition of the 225 yards to the sides *b c* and *c d* of the square, A.

The square	A	=	400 *yds.*
——	C *e f*	=	100—
——	C *g h*	=	100—
——	D	=	25—
			——
		Proof	635 *yds.*

But we are directed to "*except the right hand figure,*" and also to "*place the quotient figure on the right hand of the divisor;*" the reason of which is that the addition, *C e f* and *C g h* to the sides *bc* and *cd* of the square, A, do not leave the figure a complete square, but there is a deficiency, D, at the corner. Therefore, in dividing, the right hand figure is expected, to leave something of the dividend, for this deficiency; and as the deficiency, D, is limited by the additions, *C e f* and *C g h*, and as the quotient figure (5) is the width of these additions, consequently equal to one side of the square, D; therefore, the quotient figure (5) placed to the right hand of the divisor (4 *tens)* and multiplied into itself, gives the contents of the square, D, and the 4 *tens* = to the sum of the sides, *bc* and *cd* of the addition *Cef* and *Cgh*, multiplied by the quotient figure, (5) the width of those additions, give the contents *C e f* and *C g h*, which together subtracted from the dividend, and there being no remainder, shew that the 225 yards are disposed in the new additions *C e f*, *C g h*, and D, and the figure is seen to be continued a complete square.

Consequently, *fig.* II. shews the dimensions of a square room, 25 yards on a side, the floor of which, 625 yards of carpeting, 1 yard wide will be sufficient to cover.

The proof is seen by adding together the different parts of the figure.

Such are the principles, on which the operation of extracting the square root is grounded.

3. What is the ſquare root of 10342656? *Ans.* 3216.

4. What is the ſquare root of 43264? *Ans.* 208.

5. What is the ſquare root of 964,5192360241? *Ans.* 31,05671.

6. What is the ſquare root of 998001? *Ans.* 999.

7. What is the ſquare root of 234,09? *Ans.* 15,3.

8. What is the ſquare root of 1030892198,4001? *Ans.* 32107,51.

Supplement to the Square Root.

QUESTIONS.

1. WHAT is to be underſtood by a root? A power? The ſecond, third, and fourth powers?
2. What is the Index, or Exponent?
3. What is it to extract the Square Root?
4. Why is the given ſum pointed off into periods of two figures each?
5. In the operation, having found the firſt figure in the root, why do we ſubtract the ſquare number, that is, the ſquare of that figure, from the period in which it was taken?
6. Why do we double the root of a diviſor?
7. In dividing why do we except the right hand figure of the dividend?
8. Why do we place the quotient figure in the root and alſo to the right hand of the diviſor?
9. If there be decimals in the given number, how muſt it be pointed?
10. How is the operation of extracting the Square Root proved?

EXERCISES IN THE SQUARE ROOT.

1. A Clergyman's glebe conſiſts of three fields; the firſt contains 5 *Acr.* 2*r.* 12*p.* the ſecond, 2 *ac.* 2 *r.* 15 *p.* the third 1 *ac.* 1 *r.* 14 *p.* in exchange for which the heritors agree to give him a ſquare field equal to all the three. Sought the ſide of the ſquare? *Ans.* 39 *poles.*

2. A General has an army of 4096 men; how many muſt he place in rank and file to form them into a ſquare? *Answer* 64.

3. There is a circle whofe diameter is 4 inches, what is the diameter of a circle 4 times as large? *Ans.* 8 *inches.*

NOTE. Square the given diameter, multiply this fquare by the given proportion, and the fquare root of the product will be the diameter required. Do the fame in all fimilar cafes.

If the circle of the required diameter were to be lefs than the circle of the given diameter, by a certain proportion, then the fquare of the given diameter muft have been divided by that proportion.

4. There are two circular ponds in a gentleman's pleafure ground; the diamter of the lefs is 100 feet, and the greater is three times as large. What is its diameter? *Ans.* 173,2+

5. If the diameter of a circle be 12 inches, what will be the diameter of another circle, half fo large? *Ans.* 8, 48+*inches.*

6. A wall is 36 feet high, and a ditch before it is 27 feet wide; what is the length of a ladder, that will reach to the top of the wall from the oppoſite ſides of the ditch? *Anſwer* 45 *feet.*

NOTE. A Figure of three ſides, like that formed by the wall, the ditch and the ladder is called a *right angle triangle*, of which, the ſquare of the hypotenuſe, or ſlanting ſide, (*the ladder*) is equal to the ſum of the ſquares of the two other ſides, that is, the heighth of the wall and the width of the ditch.

7. A Line of 36 yards will exactly reach from the top of a Fort to the oppoſite bank of a river, known to be 24 yards broad; the height of the wall is required? *Anſwer* 26,83+ *yards.*

8. Glasgow is 44 miles west from Edinburgh: Peebles is exactly south from Edinburgh, and 49 miles in a straight line from Glasgow; what is the distance between Edinburgh and Peebles? *Ans.* 21,5+ *miles.*

§ 4. Extraction of the Cube Root.

TO extract the Cube Root of any number is to find another number, which multiplied into its square shall produce the given number.

RULE.

1. "Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. "Find the greatest cube in the left hand period, and put its root in the quotient.

3. "Subtract the cube thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend.*

4. "Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor.*

5. "Seek how often the divisor may be had in the dividend, and place the result in the quotient.

6. "Multiply the triple square by the last quotient figure and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the *subtrahend.*

7. "Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

NOTE. "The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root."

1. What is the cube root of 373248 ?

OPERATION.

373248(72 *the root.*
343

Divisor 14910)30248

29400
840
8
———
30248
———
00000

$7\times7\times300=14700$, *the triple square.*
$7\times30\quad=\quad210$ *the triple quotient.*
———
14910 *the divisor.*

$14700\times2=29400$
$2\times2\times210=\quad840$
$2\times2\times2\quad=\quad8$
———
30248 *the subtrahend.*

DEMONSTRATION

Of the Reason and Nature of the various steps in the operation of extracting the CUBE ROOT.

Any solid body having *six equal sides*, and each of these sides an *exact square*, is a CUBE, and the measure in length of one of its sides is the root of that cube. For if the measure in feet of any one side of such a body be multiplied three times into itself, that is, raised to the third power, the product will be the number of solid feet the whole body contains.

And on the other hand, if the cube root of any number of feet be extracted, this root will be the length of one side of a cubic body, the whole contents of which will be equal to such a number of feet.

2. Supposing a man has 13824 feet of timber, in distinct and separate blocks of one foot each ; he wishes to know how large a solid body they will make when laid together, or what will be the length of one of the sides of that cubic body ?

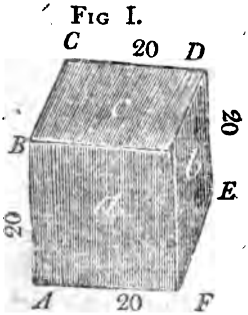
To know this, all that is necessary is to extract the cube root of that number, in doing which I propose to illustrate the operation.

OPERATION.

13824(20
8
———
5824

In this number, pointed off as the rule directs, there are two periods, of course there will be two figures in the root.

FIG I.



20
———
400
20
———
8000 *feet=the solid contents of the* CUBE.

The greatest cube in the right hand period, (13) is 8, of which 2 is the root, therefore, 2 placed in the quotient is the first figure of the root, and as it is certain we have one figure more to find in the root, we may for the present supply the place of that one figure by a cypher (20) then 20 will express the true value of that part of the root now obtained. But it must be remembered, that the *cube root* is the length of one of the sides of the cubic body, whose length, breadth, and thickness are equal. Let us then form a cube, *Fig.* I. each side of which shall be supposed 20 feet ; now the side A. B. of this cube, or either of the sides, shews the root, (20) which we have obtained.

The Rule next directs, *subtract the cube, thus found, from the said period and to the remainder bring down the next period, &c.* Now this cube (8) is the solid contents of the figure we have in representation. Made evident thus—Each side of this figure is 20, which being raised to the 3d power, that is, the length, breadth and thickness being multiplied into each other, gives the solid contents of that figure=8000 feet. And the cube of the root, (2) which we have obtained is 8, which placed under the period from which it was taken as it falls in the place of *thousands*, is 8000, equal to the solid contents of the cube A B C D E F, which being subtracted from the given number of feet, leaves 5824 feet.

Hence *Fig.* I, exhibits the exact progress of the operation. By the operation 8000 *ft.* of the timber are disposed of, and the figure shews the disposition made of them, into a square solid pile which measures 20 feet on every side.

Now this figure or pile is to be enlarged by the addition of the 5824 feet, which remains; and this addition must be so made, that the figure or pile, shall continue to be a complete cube, that is have the measure of all its sides equal.

To do this the addition must be made equally to the three different squares, or faces *a*, *c* and *b*.

The next step, in the operation is, to find a divisor; and the proper divisor will be, the number of square feet contained in all the points of the figure, to which the addition of the 5824 feet is to be made.

Hence we are directed "*multiply the square of the quotient by* 300," the object of which is, to find the superficial contents of the three faces, *a*, *c*, *b*, to which the addition is now to be made. And that the square of the quotient, multiplied by 300 gives the superficial contents of the faces *a*, *c*, *b*, is evident from what follows.

Side A B=20 ⎫
Side A F=20 ⎬ *of the face*, *a*.
——
Superficial contents=400 ⎭
3
——
The triple square 1200=*the superficial contents of the faces*, *a*, *c*, *and b*.

The two sides A B and A F of the face, *a*, multiplied into each other, give the superficial content of *a*, and as the faces, *a*, *c*, and *b*, are all equal, therefore, the content of the face, *a* multiplied by 3, will give the contents of *a*, *c*, and *b*.

2 *quotient figure.*
2
——
4 *the square of* 2
300
——
The triple square of 1200=*the superficial contents of the faces a*, *c*, *and b*.

Here the quotient figure 2, is properly, *two tens*, for there is another figure to follow it in the root, and the square of 2, standing as *units*, is 4, but its true value is 20 (*the side* A B) of which the square is 400, we therefore lose two cyphers, and these two cyphers are annexed to the figure 3.—Hence it appears, that we square the quotient, with a view to find the superficial content of the face, or square *a*; we multiply the square of the quotient by 3, to find the superficial contents of the three squares, *a*, *c*, and *b*, and two cyphers are annexed to the 3, because in the square of the quotient *two cyphers* were lost, the quotient requiring a cypher before it in order to express its true value which would throw the quotient (2) into the place of *tens*, whereas now it stands in the place of units.

Now when the additions are made to the squares *a*, *c*, and *b*, there will evidently be a deficiency, along the whole length of the sides of the squares between each of the additions, which must be supplied before the figure can be a complete cube. These deficiencies will be 3, as may be seen, *Fig*. II. *n*. *n*. *n*.

Therefore it is, that we are directed, "*multiply the quotient by* 30 *calling it the triple quotient.*"

The triple quotient is the sum of the three lines, or sides against which are the deficiencies, *n, n, n,* all which meet at a point, nigh the centre of the figure. This is evident from what follows.

The deficiencies are 3 in number, they are the whole length of the sides; the length of each side is 20 feet, therefore 20
3

Triple quotient 60 = *to the length of* 3 *sides where are deficiencies to be filled.*

2 *quotient.*
30

Triple quotient 60 *equal to the length of* 3 *sides, &c.*

Here, as before, the quotient lacks a cypher to the right hand to exhibit its true value; the quotient, *itself,* is the length of one of the sides, where are the deficiencies; it is multiplied by 3, because there are 3 deficiencies, and a cypher is annexed to the 3 because it has been omitted in the quotient, which gives the same product, as if the true value of the quotient, 20, had been multiplied by 3 alone.

We now have { 1200 the triple square.
60 the triple quotient.

The sum of which, 1260 is the divisor, equal the number of square feet contained, in all the points of the figure or pile, to which the addition of the 5824 feet is to be made.

OPERATION *continued.*

13824(24 *the root.*
8
Divis. 1260)5824 *the dividend.*
4800
960
64
5824 *subtrahend.*
0000

This figure in the root, (4) shews the depth of the addition, on every point where it is to be made to the pile or figure, represented, *Fig.* I.

FIG. II

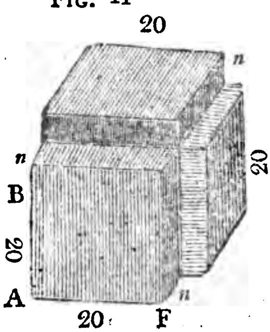


FIG. II. exhibits the additions made to the squares *a, c, b,* by which they are covered or raised by a depth of 4 feet.

The next step in the operation is to find a subtrahend which subtrahend is the number of solid feet contained in all the additions to the cube, by the last figure 4.

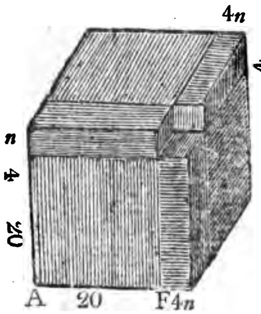
Therefore, the rule directs, "*multiply the triple square by the last quotient figure.*"

The triple square, it must be remembered, is the superficial contents of the faces *a, c,* and *b,* which multiplied by 4, the depth now added to those faces, or squares, gives the number of solid feet contained in the additions by the last quotient figure 4.

1200 *triple square.*
4 *last quotient figure.*

4800 *feet, equal to the addition made to the squares, or faces, a, c, b, of Fig. I. a depth of 4 feet on each.*

Fig. III.



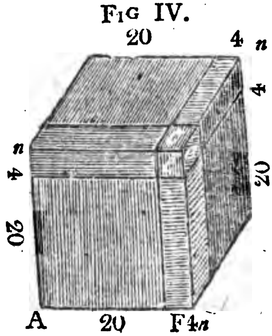
Then, "*multiply the ſquare of the laſt quotient figure by the triple quotient.*" This is to fill the deficiencies, *n*, *n*, *n*, *Fig.* II. Now theſe deficiencies are limited in length, by the length of the ſides (20) and the triple quotient is the ſum of the length of the deficiencies. They are limited in width by the laſt quotient figure (4) the ſquare of which gives the area, or ſuperficial contents at one end, which multiplied into their length, or the triple quotient, which is the ſame thing, gives the contents of thoſe additions 4*n*4, 4*n*, 4*n*.

60 *triple quotient.*
16 *ſquare of the laſt quotient figure, Fig. III.*

360
60

960 *feet diſpoſed in the deficiencies, between the additions to the ſquares, a, c, b, Fig. III. exhibits theſe deficiencies ſupplied, 4n4, 4n, 4n, and diſcovers another deficiency where theſe approach together, of a corner wanting to make the figure a complete cube.*

Fig IV.



Laſtly, "*cube the laſt quotient figure.*" This is done to fill the deficiency *Fig.* III. left at one corner, in filling up the other deficiencies, *n*, *n*, *n*. This corner is limited by thoſe deficiencies on every ſide, which were 4 feet in breadth, conſequently, the ſquare of 4 will be the ſolid content of the corner, which in *Fig.* IV. *e*, *e*, *e*, is ſeen filled.

4
4

16
4

64 *feet diſpoſed in the corner, e, e, e, where the additions n, n, n, approach together.*

Now the ſum of theſe additions make the ſubtrahend, which ſubtract from the dividend, and the work is done.

Figure IV. ſhews the pile which 13824 ſolid blocks of one foot each, would make when laid together. The root (24) ſhews the length of a ſide. *Fig.* I. ſhews the pile which would be formed by 8000 of thoſe blocks, firſt laid together: *Fig. II.* and *Fig. III.* ſhews the changes which the pile paſſes through in the addition of the remaining 5824 blocks or feet.

Proof. By adding the contents of the firſt figure, and the additions exhibited in the other figures together.

Feet.	
8000	*Contents of Fig. I.*
4800	*addition to the faces or ſquares a, c, and b, Fig. II.*
960	*addition to fill the deficiencies n, n, n, Fig III.*
64	*addition at the corner e, e, e, Fig. IV. where the additions which fill the deficiencies n, n, n, approach together.*
13824	Number of blocks or ſolid feet, all which are now diſpoſed in Fig. IV. forming a pile, or ſolid body of timber, 24 feet, on a ſide.

Such is the demonſtration of the reaſon and nature of the various ſteps in the operation of extracting the cube root. Proper views of the figures, and of thoſe ſteps in the operation illuſtrated by them, will not generally be acquired without ſome diligence or attention. Scholars, more eſpecially will meet with difficulty. For their aſſiſtance, ſmall blocks might be formed of wood in imitation of the Figures, with their parts in different pieces. By the help of theſe, Maſters, in moſt inſtances, would be able to lead their pupils into right conceptions of thoſe views, which are here given of the nature of this operation.

3. What is the cube root of 21024576 ? *Ans.* 276.

4. What is the cube root of 253395799552? *Answer*, 6328.

5. What is the cube root of 84,604519 ? *Answer*, 4,39.

6. What is the cube root of 2 ? *Answer*, 1,25+

Supplement to the Cube Root.

QUESTIONS.

1. WHAT is a cube ?
2. What is underſtood by the cube root ?
3. What is it to extract the cube root ?
4. In the operation having found the firſt figure of the root, why is the cube of it ſubtracted from the period in which it was taken ?
5. Why is the ſquare of the quotient multiplied by 300 ?
6. Why is the quotient multiplied by 30 ?
7. Why do we add the triple ſquare and the triple quotient together, and the ſum of them call the diviſor ?
8. To find a ſubtrahend, why do we multiply the triple ſquare by the laſt quotient figure ? The ſquare of the laſt quotient figure by the triple quotient ? Why do we cube the quotient figure ? Why do theſe ſums added, make the ſubtrahend ?
9. How is the operation proved ?

EXERCISES IN THE CUBE ROOT.

1. If a bullet 6 inches diameter weigh 32lb. what will a bullet of the ſame metal weigh, whoſe diameter is 3 inches ? *Ans.* 4*lb.*

Note. "The ſolid contents of ſimilar figures are in proportion to each other, as the cubes of their ſimilar ſides, or diameters."

2. What is the ſide of a cubical mound equal to one 288 feet long, 216 broad, and 48 high? *Ans.* 144 *feet.*

3. There is a cubical veſſel, whoſe ſide is 2 feet: I demand the ſide of a veſſel, which ſhall contain three times as much? *Ans.* 2 *feet* 10 *inches and* $\frac{3}{5}$ *nearly.*

Note. Cube the given ſide, multiply it by the given proportion, and the cube root of the product will be the ſide ſought.

§ 5. Fellowſhip.

FELLOWSHIP is a rule by which merchants and others, trading in partnerſhip, compute their particular ſhares of the gain or loſs, in proportion to their ſtock and the time of its continuance in trade.

It is of two kinds, *Single and Double.*

Single Fellowſhip,

Is when the ſtocks are employed equal times.

RULE.

As the whole ſum of the ſtocks is to the whole gain or loſs, ſo is each man's particular ſtock to his particular ſhare of the gain or loſs.

PROOF. Add all the ſhares of the gain or loſs together; and, if the work be right, the ſum will be equal to the whole gain or loſs.

EXAMPLES.

1. Two merchants, A and B, make a joint ſtock of 200 dollars: A puts in 75 dollars, and B 125 dollars; they trade and gain 50 dollars. What is each man's ſhare of the gain?

OPERATION.

```
   Dolls. Dolls.  Dolls.
As 200  :  50  : : 75            As 200 : 50 : : 125
           75                                   125
          ----                                 ----
          250                                   250
         350                                   100
       -------D. cts.                           50
    200)3750( 18,75 A's ſhare.               -------D. cts.
        200                              200)6250(31,25 B's ſhare.
       -----                                 600
        1750                                 ---
        1600                                 250
        ----                                 200
         1500                                ---
         1400                                 500
         ----                                 400
          1000      18,75 A's ſhare.         ----
          1000      31,25 B's ſhare.          1000
          ----      -----                     1000
                    50,00 proof.
```

2. Divide the number 360 into 4 ſuch parts, which ſhall be to each other as 3, 4, 5, and 6.

```
 60 ⎫
 80 ⎬ Anſwer.
100 ⎪
120 ⎭
---
360 Proof.
```

3. A man died leaving 3 sons, to whom he bequeathed his estate in the following manner, viz. to the eldest he gave 184 dollars, to the second 155 dollars, and to the third 96 dollars; but when his debts were paid, there were but 184 dollars left: What is each one's proportion of his estate?

Ans. 77,829 }
65,563 } *shares.*
40,606 }

4. A and B companied:—A put in £45, and took $\frac{3}{5}$ of the gain: what did B put in? *Ans.* £30.

Double Fellowship.

DOUBLE FELLOWSHIP, or Fellowſhip with time, is when the ſtocks of partners are continued unequal times.

RULE.

Multiply each man's ſtock by the time it was continued in trade. Then, as the whole ſum of the products is to the whole gain or loſs, ſo is each man's particular product to his particular ſhare of the loſs or gain.

EXAMPLES.

1. A, B and C, entered into partnerſhip: A put in 85 dollars for 8 months; B put in 60 dollars for 10 months; and C put in 120 dollars for 3 months; by misfortune they loſt 41 dollars: What muſt each man ſuſtain of the loſs?

OPERATION.

```
 85      60     120      680 A's product.
  8      10       3      600 B's product.
---     ---     ---      360 C's product.
680     600     360      ----
                         1640
```

```
As 1640 : 41 : : 680            As 1640 : 41 : : 600
              680                             600
              ---                             ---
              680               164|0)2460|0(15 B's loſs.
            2720                      164
            ----                      ---
164|0)2788|0(17 A's loſs.              820
      164                              820
      ---                              ---
      1148
      1148
      ----
      0000
```

```
As 1640 : 41 : : 360            Dolls.
              360                17 A's loſs.
              ---                15 B's loſs.
             2460                 9 C's loſs.
             123                 --
            ----                 41 Proof.
164|0)1476|0(9 C's loſs.
      1476
      ----
      0000
```

2. A, B, and C, trade together: A, at first put in 480 dollars for 8 months, then put in 200 dollars more, and continued the whole in trade 8 months longer; at the end of which he took out his whole stock: B put in 800 dollars for 9 months, then took out $583,333 and continued the rest in trade 3 months, C put in $366,666 for 10 months, then put in 250 dollars more, and continued the whole in trade 6 months longer. At the end of their partnership, they had cleared 1000 dollars; what is each man's share of the gain?

Ans. Dolls. 378,827 A's share.
—— 320,452 B's share.
—— 300,721 C's share.

Supplement to Fellowship.

QUESTIONS.

1. WHAT is Fellowſhip ?
2. Of how many kinds is Fellowſhip ?
3. What is ſingle Fellowſhip ?
4. What is the rule for operating in Single Fellowſhip ?
5. What is double Fellowſhip ?
6. What is the rule for operating in double Fellowſhip ?
7. How is Fellowſhip proved ?

EXERCISES IN FELLOWSHIP.

A, B, and C, hold a paſture in common for which they pay £20 per annum. In this paſture, A had 40 oxen for 76 days ; B had 36 oxen for 50 days, and C had 50 oxen for 90 days. I demand what part each of theſe tenants ought to pay for the £20 ?

	£.	s.	d.	qr.	
Ans.	6	10	2	$1\frac{2340}{9340}$	A's part.
	3	17	1	$0\frac{2600}{9340}$	B's part.
	9	12	8	$2\frac{5[illegible]00}{9340}$	C's part.

§ 6. Barter.

BARTER is the exchanging of one commodity for another, and teaches merchants so to proportion their quantities, that neither shall sustain loss.

PROOF. By changing the order of the question.

RULE.

1. *When the quantity of one commodity is given, with its value, or the value of its integer, as also the value of the integer of some other commodity to be exchanged for it, to find the quantity of this commodity:* Find the value of the commodity of which the quantity is given, then find how much of the other commodity at the rate proposed, may be had for that sum.

2. *If the quantities of both commodities be given, and it should be required to find how much of some other commodity, or how much money should be given, for the inequality of their values:* Find the separate value of the two given commodities, subtract the less from the greater, and the remainder will be the balance, or value of the other commodity.

3. *If one commodity is rated above the ready money price, to find the bartering price of the other:* Say, as the ready money price of the one is to the bartering price so is *that* of the other to *its* bartering price.

EXAMPLES.

1. How much coffee, at 25 cents per lb. can I have for 56 lb. of tea at 43 cents per lb.?

OPERATION.

```
       5 6 lb. of tea.
       ,4 3 per lb.
     -------
      1 6 8
    2 2 4
    ---------lb. oz.
2 5 )2 4,0 8(96 5 3/25 answer.
     2 2 5
    -------
      1 5 8
      1 5 0
     -------
          8
        1 6
     -------
  2 5)1 2 8(5
      1 2 5
     -------
          3
```

2. I have 760 gallons of molasses at 37 cents, 5 mills, per gallon, which I would exchange for 66 Cwt. 2qr. of cheese, at 4 dollars per Cwt. Must I pay or receive money and how much?

Ans. must receive 19 *dolls.*

3. A and B, barter ; A has 150 bushels of wheat at 5s. 9d. per bushel, for which B gives 65 bushels of barley, worth 2s. 10d. per bushel, and the balance in oats at 2s. 1d. per bushel ; what quantity of oats must A receive from B ? *Answer*, $325\frac{1}{25}$ *bushels.*

4. A has linen cloth worth 20d. an Ell, ready money ; but in barter he will have two shillings ; B has broadcloth worth 14s. 6d. per yard ready money ; at what price ought the bread cloth to be rated in barter ?

Answer, 17*s.* 4*d.* 3*q.* $\frac{4}{20}$ *per yard.*

Supplement to Barter.

QUESTIONS.

1. WHAT is Barter?
2. When and how does this rule become useful to merchants?
3. When a given quantity of one commodity is bartered for some other commodity, how is the quantity that will be required of this last commodity found?
4. If the quantity of both commodities be given and it be required to know how much of some other commodity, or how much money must be given for the inequality, what is the method of procedure?
5. If one commodity be rated above the money price, how do you proceed to find the bartering price of the other commodity?
6. How is Barter proved?

EXERCISES.

1. A and B bartered: A had 41 Cwt. of hops, 30s. per Cwt. for which B gave him £20 in money, and the rest in prunes at 5d. per lb. I demand how many prunes B gave A besides the £20? *Ans.* 17 *C.* 3*qrs.* 4*lb.*

2. How much wine, at \$1,28 per gallon, must I have for 26 Cwt. 2qr. 14lb. of raisins, at \$9,444 per Cwt. *Ans.* 196 *gal.* 1*qt.* 1*pt. and* $\frac{1}{2}$ *very nearly.*

§ 7. Loss and Gain.

" LOSS and GAIN is a rule which enables merchants to eſtimate their profit or loſs, in buying and ſelling goods ; alſo to raiſe or fall the price of them, ſo as to gain, or loſe ſo much per cent."

CASE 1.

To know what is gained or loſt per cent. Firſt, find what the gain or loſs is by ſubtraction ; then, as the price it coſt is to the gain or loſs, ſo is 100 dollars (or £100) to the gain or loſs, per cent.

EXAMPLES.

1. If I buy candles at 16 cents, 7 mills per lb. and ſell them at 20 cents per lb. what ſhall I gain per cent. or in laying out 100 dollars ?

OPERATION.

I ſell at ,20 per lb.
bought at ,167 per lb.

I gain ,033 per lb.

Then, as 167 : ,0 3 3 : : 100
1 0 0
D. cts.
,167(3, 3 0 0(19,76 *Ans.*
1 6 7
1 6 3 0
1 5 0 3
1 2 7 0
1 1 6 9
1 0 1 0
1 0 0 2
8

2. Bought indigo, at $1,20 per lb. and ſold the ſame at 90 cents per lb. what was the loſs per cent. ?

Ans. 25 *dollars.*

3. Bought 37 gallons of Brandy, at $1,10 per gallon, and ſold it for $40 ; what was gained or loſt per cent ? *Ans.* $1,719 *loſs.*

4. Bought hats at 4s. a piece, and ſold them again at 4s9 ; what is the profit in laying out £100 ?

Ans. £18,15*s.*

CASE 2.

To know how a commodity must be sold to gain or lose so much per cent. As 100 dollars *(or £100)* is to the price ; so is 100 dollars *(or £100)* with the profit added, or loss subtracted, to the gaining or losing price.

EXAMPLES.

1. If I buy wheat at $1,25 per bushel, how must I sell it to gain 15 per cent ?

OPERATION.

```
As 100 : 1,2 5 : : 115
           1 1 5
          -------
           6 2 5
         1 2 5
       1 2 5
      ----------D.cts.m.
100)1 4 3, 7 5(1,43 7 Ans.
    1 0 0
    -----
      4 3 7
      4 0 0
      -----
        3 7 5
        3 0 0
        -----
          7 5 0
          7 0 0
          -----
            5 0
```

2. If a barrel of rum cost 15 dollars, how must it be sold to lose 10 per cent ? *Ans.* $13,50.

3. If 120 lb. of steel cost £7 how must I sell it per lb. to gain £15½ per cent ? *Ans.* 1s4 *per lb.*

Supplement to Loss and Gain.

QUESTIONS.

1. What is Loſs and Gain ?
2. Having the price at which goods are bought and ſold, how is the Loſs or Gain eſtimated ?
3. To know how much a commodity muſt be valued at to gain or loſe ſo much per cent. what is the method of procedure ?
4. How may queſtions in Loſs and Gain be proved ?

EXERCISES.

1. A draper bought 100 yards of broadcloth for £56. I demand how he muſt ſell it per yard, to gain £15 in laying out £100 ? *Ans.* 12*s.* 10*d.* 2*q.* $\frac{20}{100}$

2. Bought 30 hogſheads of molaſſes, at 600 dollars ; paid in duties $20,66 ; for freight $40,78 ; for porterage $6,05 and for inſurance, $30,84. If I ſell it at 26 dollars per hogſhead, how much ſhall I gain per cent. ? *Ans.* $11,695.

§ 8. Duodecimals;

OR

CROSS MULTIPLICATION.

THIS rule is particularly useful to Workmen and Artificers in casting up the contents of their work.

Dimensions are taken in feet, inches and parts. Inches and parts are sometimes called primes ('), seconds ("), thirds (‴), and fourths (⁗).

TABLE.

12	*Fourths make*	1	*Third.*
12	*Thirds,* —	1	*Second.*
12	*Seconds,* —	1	*Inch or Prime.*
12	*Inches or Pr.*	1	*Foot.*

By this rule also may be calculated the solid contents of bodies, having the measures of their different sides, and is very useful, therefore, in measuring wood.

RULE.

1. Under the multiplicand write the corresponding denominations of the multiplier.

2 Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write the result of each under its respective term, observing to carry an unit for every 12, from each lower denomination to its superior.

3. In the same manner multiply the multiplicand by the inches in the multiplier, and write the result of each term in the multiplicand thus multiplied, *one place* to the right hand in the product.

4. Proceed in the same manner with the other parts in the multiplier, which if seconds, write the result *two places* to the right hand; if thirds, *three places*, &c. and their sum will be the answer required.

The more easily to comprehend the rule, NOTE. *Feet multiplied by Feet give feet.—Feet multiplied by Inches give Inches.—Feet multiplied by Seconds give Seconds.—Inches multiplied by Inches give Seconds.—Inches multiplied by Seconds give Thirds.—Seconds multiplied by Seconds give Fourths.*

EXAMPLES.

1. Multiply 7 feet, 3 inches, 2 Seconds, by 1 foot, 7 inches, and 3 Seconds.

OPERATION.

	F.	*I.*	"		
	7	3	2		
	1	7	3		
	7	3	2	‴	
	4	2	10	2	⁗
		1	9	9	6
Prod.	11	7	9	11	6

Here I multiply the 7*f.* 3*in.* 2" by the 1*f.* in the multiplier, which gives seconds, inches and feet.

Next I multiply the same 7*f.* 3*in.* 2" by the 7*in.* saying 7 times 2 is 14 which is once 12 and 2 over, which (2) I set down one place to the right hand, that is in the place of thirds, and carry 1 to the next place, and proceed in the same manner with the other terms. Lastly I multiply the multiplicand by the 3" saying 3 times 2 is 6 which I set down two places to the right hand and so proceed with the other terms of the multiplicand. The sum of all the products is the answer.

2		3		4	
F. I.		F. I		F. I.	
7 5	F. I. ″	4 6	F. I.	8 3	F. I.
3 . 9	27 9 9 *Prod.*	5 8	25 6 *Prod.*	6 4	52 3 *Prod.*

5. Multiply 7*f.* 1*in.* 9″ by 7*f.* 8*in.* 9″
Product 55*f.* 2*in.* 9″ 8‴ 9⁗

6. Multiply 9*f.* 8*in.* 7″ by 12*f.* 3*in.* 10″
Product 119*f.* 8′ 2″ 10‴ 10⁗

7. How much wood in a load which meafures 10*f.* in length, 3*f.* 9*in.* in width, and 4*f.* 8*in.* in height; and how much will it coft at 1 *dol.* 33*cts.* per cord? *Ans.* 1 *cord, and* 47 *folid feet over; it will coft* 1 *dol.* 81 *cts.* 8*m.*

Or, we may multiply by the feet as already directed, and for the inches, take such parts of the multiplicand, &c. as the inches are aliquot or even parts of a foot, as done in the rule of Practice.

8. How many square feet in a board of 16 feet, 4 inches in length, and 2 feet, 8 inches wide?

OPERATION

	Ft.	in.	''
6 *inches is* $\frac{1}{2}$	16	4	
	2	8	
	32	8	
2————$\frac{1}{3}$	8	2	
	2	8	8
Ans.	43	6	8

Here, in the first place I multiply the 16*ft.* 4*in.* by the feet (2) of the multiplier; the inches (8) not being an even part of a foot, I take such as are an even part; thus, 6*in.* is half a foot, therefore divide the multiplicand by 2 for 6 inches, and that quotient by 3, (2*in. is* $\frac{1}{3}$ *of* 6 *inches*) for 2 inches, all which being added, give the product of 16 feet, 4 inches multiplied by 2*ft.* 8*in.*

9. Another board is 18 feet 9 inches in length, and 2 feet, 6 inches wide, how many square feet does it contain? *Ans.* 46*f.* 10*in.* 6''.

By Practice. *By Duodecimals.*

10. There is a stock of 15 boards, 12 feet 8 inches in length, and 13 inches wide; how many feet of boards does the stock contain?

Ans. 205 *feet* 10 *inches.*

By Practice. *By Duodecimals.*

Supplement to Duodecimals.

QUESTIONS.

1. OF what uſe are Duodecimals ? To whom more eſpecially are they uſeful ?
2. In what are dimenſions taken ?
3. How do you proceed in the multiplication of Duodecimals ?
4. For what number do you carry ?
5. What do you obſerve in regard to ſetting down the product different from what is common in the multiplication of other numbers ?
6. Of what term is the product which ariſes from the multiplication of feet by inches ? Feet by ſeconds ? Inches by inches ? Inches by ſeconds ? Seconds by ſeconds ?
7. In what way can the operation be varied ?

EXERCISES.

1. Multiply 76 feet 3 inches 9 ſeconds, by 84 feet 7 inches 11 ſeconds.

OPERATION.

	F.	I.	″			
6 inches is $\frac{1}{2}$)	76	3	9			
	84	7	11			
76 × 4 =	304	0	0			
76 × 8 =	608	0	0			
3 × 84 =	21	0	0			
9 × 84 =	5	3	0	‴		
I. 1 $\frac{1}{8}$)	38	1	10	6		
″6 $\frac{1}{2}$)	6	4	3	9	⁗	
3 $\frac{1}{2}$) and 2 $\frac{1}{3}$)	3	2	1	10	6	
	1	7	0	11	3	
	1	0	8	7	6	
Prod.	6460	7	1	8	3	

2. What is the product of 371 feet 2 inches 6 ſeconds, multiplied by 181*f.* 1*in.* 9″.
Ans. 67242*f.* 10*in.* 1″ 4‴ 6⁗

3. How many ſquare feet in a ſtock of 12 boards, 17*f.* 7′ long, and, 1*f.* 5*in.* wide ? *Ans.* 298*f.* 11′.

4. How many cubic feet of wood in a load 6*f.* 7′ long, 3*f.* 5′ high, and 3*f.* 8′ wide ?
Ans. 82*f.* 5′ 8″ 4‴

The Dimenſions of Wainſcotting, Paving, Plaſtering, and Painting, are taken in Feet and Inches, and the contents given in Yards.

PAINTERS AND JOINERS.

To find the dimenſions of their work, take a line and apply one end of it to any corner of the room, then meaſure the room going into every corner with the line, till you come to the place where you firſt began; then ſee how many feet and inches the ſtring contains; this call the *Compaſs* or round, which multiplied into the height of the room, and the product divided by 9, the quotient will be the contents in yards.

EXAMPLES.

1. If the height of a room painted be 12*f.* 4*in.* and the compaſs 84*f.* 11*in.* how many ſquare yards does it contain? *Ans.* 116 *Y.* 3*f.* 3′ 8″

2. There is a room wainſcotted the compaſs of which is 47*f.* 3′ and the height 7*f.* 6′. What is the content in ſquare yards? *Ans.* 39 *Y.* 3*f.* 4′ 6″

GLAIZERS' WORK BY THE FOOT.

To find the dimenſions of their work, multiply the height of windows by their breadth.

EXAMPLES.

There is a houſe with 4 tiers of windows, and 4 windows in a tier; the height of the firſt tier is 6*f.* 8′; of the ſecond 5*f.* 9′; of the third, 4*f.* 6′; and of the fourth, 3*f.* 10′; and the breadth of each is 3*f.* 5′; What will the glazing come to at 19 cents per foot? *Ans.* $53,88.

§ 9. Alligation.

ALLIGATION is the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality. It is of two kinds, MEDIAL and ALTERNATE.

ALLIGATION MEDIAL.

Alligation Medial is when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

RULE.

As the sum of the quantities or whole composition is to their total value so is any part of the composition to its value or mean price.

EXAMPLES.

1. A Farmer mingled 19 bushels of wheat at 6*s.* per bushel, and 40 bushels of rye, at 4*s.* per bushel, and 12 bushels of barley, at 3*s.* per bushel together, I demand what a bushel of this mixture is worth?

OPERATION.

	Bush.		*s.*	*£*	*s.*
	19	Wheat, at	6 is	5	14
	40	Rye, —	4 —	8	
	12	Barley, —	3 —	1	16
Sum of the simples	71	Total value		15	10

Bush. *£.* *s.* *Bush.*
As 71 : 15 10 :: 1
20
71)310(4*s.* 4*d.* $1\frac{41}{71}$*q.* *Ans.*
284
26
12
)312(4*d.*
284
28
4
)112(1*qr.*
71
41

2. A refiner having 5lb. of silver bullion, of 8oz. fine, 10lb. of 7oz. fine, and 15lb. of 6oz. fine, would melt all together; I demand what fineness 1lb. of this mass shall be?

Ans. 6*oz.* 13*pwts.* 8*grs. fine.*

ALLIGATION ALTERNATE,

Is the method of finding what quantity of any number of simples, whose rates are given will compose a mixture of a given rate ; it is, therefore, the reverse of Alligation Medial, and may be proved by it.

RULE.

1. Write the prices of the simples, the least uppermost, &c. in a column under each other.

2. Connect with a continued line the price of each simple or ingredient, which is less than that of the compound, with one or any number of those that are greater than the compound, and each greater rate or price with one or any number of those that are less.

3. Write the difference between the mean rate or price and that of each of the simples, opposite to the rates with which they are connected.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate, but if there be several, their sum will be the quantity.

Note. Questions in this rule admit of as many various answers as there are various ways of connecting the rates of the ingredients together.

EXAMPLES.

1. A goldsmith would mix gold of 18 carats fine, with some of 16, 19, 22 and 24 carats fine, so that the compound may be 20 carats fine ; what quantity of each must he take ?

			OPERATION. oz. car. *fine.*		PROOF.
Mix 20 car.	16	4	4 of gold 16	Ans.	16×4=64
	18	2	2 — 18		18×2=36
	19	2	2 — 19		19×2=38
	22	2×1	3 — 22		22×3=66
	24	4	4 — 24		24×4=96
			15 — 20 *carats fine*		oz. 15)300(20 *car. fine.*

2. A Druggist had several sorts of Tea, viz. one sort at 12*s.* ber lb. another sort at 11*s.* a third at 9*s.* and a fourth at 8*s.* per lb. I demand how much of each sort he must mix together, that the whole quantity may be afforded at 10*s.* per lb.

	lb.	*s.p.lb.*		*lb.*	*s.p.lb.*		*lb.*	*s.p.lb.*
1 Ans.	2 at	12	2 Ans.	3 at	12	3 Ans.	1 at	12
	1 at	11		2 at	11		2 at	11
	1 at	9		2 at	9		2 at	9
	2 at	8		3 at	8		1 at	8
	lb.	*s.p.lb.*		*lb.*	*s.p.lb.*		*lb.*	*s.p.lb.*
4 Ans.	1 at	12	5 Ans.	3 at	12	6 Ans.	2 at	12
	3 at	11		1 at	11		3 at	11
	3 at	9		3 at	9		1 at	9
	1 at	8		2 at	8		3 at	8

7 Ans. 3 lb. of each sort.

Note. These *seven answers* arise from as many different ways of linking the *rates* of the simples together.

CASE 2.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixture are given to find the ſeveral quantities of the reſt in proportion to the given quantity, *take the difference between each price and the mean rate as before.* Then ſay,

As the difference of that ſimple whoſe quantity is given,
Is to the given quantity,
So is the reſt of the differences *ſeverally* ;
To the ſeveral quantities required.

EXAMPLES.

1. How much wine, at 80 cents, at 88, and 92 cents per gallon, muſt be mixed with four gallons of wine at 75 cents per gallon, ſo that the mixture may be worth 86 cents per gallon?

OPERATION.

$$86\begin{cases}75 \\ 80 \\ 88 \\ 92\end{cases}\qquad \begin{array}{l} 6+2=8 \text{ ſtands againſt the given quantity.} \\ 2+6=8 \\ 6+11=17 \\ 11+6=17 \end{array}$$

$$\text{As } 8 : 4 :: \begin{cases} 8 : 4 \;\text{ at } 80 \\ 17 : 8\tfrac{1}{2} \text{ — } 88 \text{ per. gal. The anſwer.} \\ 17 : 8\tfrac{1}{2} \text{ — } 92 \end{cases}$$

(gal. — cts.)

2. A man being determined to mix 10 buſhels of wheat at 4*s.* per buſhel, with rye at 3*s.* with barley at 2*s.* and with oats at 1*s.* per buſhel ; I demand how much rye, barley, and oats muſt be mixed with the 10 buſhels of wheat that the whole may be ſold at 28d. per buſhel.

1 Ans. { *B. p.* 2 2 of Rye; 5 0 of Barley; 12 2 of Oats }

2 Ans. { *B.* 40 of Rye; 50 of Barley; 20 of Oats }

3 Ans. { *B.* 8 of Rye; 10 of Barley; 14 of Oats }

4 Ans. { *B.* 15 of Rye; 11 of Barley; 14 of Oats }

5 Ans. { *B. p.* 12 2 of Rye; 5 0 of Barley; 17 2 of Oats }

6 Ans. { *B.* 2 of Rye; 14 of Barley; 10 of Oats }

7 Ans. { *B.* 50 of Rye; 70 of Barley; 20 of Oats }

CASE 3.

When the rates of the feveral ingredients, the quantity to be compounded, and the mean rate of the whole mixture, are given to find how much of each fort will make up the quantity ; *find the difference between the mean rate, &c. as in cafe* 1. *Then,*

As the fum of the quantities, or differences,
Is to the given quantity or whole compofition ;
So is the difference of each rate,
To the required quantity of each rate.

EXAMPLES.

1. How many gallons of water of no value, muft be mixed with brandy, at one dollar twenty cents per gallon, fo as to fill a veffel of 75 gallons, that may be afforded at 92 cents per gallon ?

OPERATION.

$$92\left\{\begin{array}{rr} \textit{Gal.} & \\ 0 & 28 \\ 1,20 & 92 \\ \hline \text{Sum.} & 120 \end{array}\right. \qquad \begin{array}{c} \textit{Gal.} \quad \textit{Gal.} \\ \text{As } 120 : 75 :: \end{array} \left\{\begin{array}{rcl} \textit{Gal.} & & \textit{Gal.} \\ 28 & : & 17\tfrac{1}{2} \text{ of Water} \\ 92 & : & 57\tfrac{1}{2} \text{ of Brandy} \\ \hline & & 75 \text{ given quantity.} \end{array}\right.$$

2. Suppofe I have 4 forts of currants, of 8*d.* 12*d.* 18*d.* and 22*d.* per lb. of which I would mix 120lb. and fo much of each fort as to fell them at 16*d.* per lb. how much of each muft I take ?

$$\text{Ans.}\left\{\begin{array}{rcr} \textit{lb.} & \text{at} & \textit{d.} \\ 36 & — & 8 \\ 12 & — & 12 \\ 24 & — & 18 \\ 48 & — & 22 \end{array}\right\}\text{per lb.}$$

3. A grocer has currants of 4*d.* 6*d.* 9*d.* and 11*d.* per lb. and he would make a mixture of 240 lb. fo that it might be afforded at 8*d.* per lb. how much of each fort muft he take ?

$$\text{Ans.}\left\{\begin{array}{rcr} \textit{lb.} & \text{at} & \textit{d.} \\ 72 & — & 4 \\ 24 & — & 6 \\ 48 & — & 9 \\ 96 & — & 11 \end{array}\right\}\text{per lb.}$$

Supplement to Alligation.

QUESTIONS.

1. What is Alligation ?
2. Of how many kinds is Alligation ?
3. What is Alligation MEDIAL ?
4. What is the rule for operating ?
5. What is Alligation ALTERNATE ?
6. When a number of ingredients of different prices are mixed together, how do we proceed to find the mean price of the compound or mixture ?
7. When one of the ingredients is limited to a certain quantity, what is the method of procedure ?
8. When the whole compofition is limited to a certain quantity, how do you proceed ?
9. How is Alligation proved ?

EXERCISES.

1. A Grocer would mix three forts of fugar together ; one fort at 10*d.* per lb. another at 7*d.* and another at 6*d.* how much of each fort muft he take that the mixture may be fold for 8*d.* per lb. ?
Ans. 3*lb. at* 10*d.* 2 *at* 7*d. and* 2 *at* 6*d.*

2. A Goldfmith has feveral forts of gold ; fome of 24 carats fine, fome of 22, and fome of 18 carats fine, and he would have compounded of thefe forts the quantity of 60 oz. of 20 carats fine ; I demand how much of each fort he muft have ?
Ans. 12 *oz.* 24 *carats fine,* 12 *at* 22 *carats fine, and* 36 *at* 18 *carats fine.*

§ 10. Position.

POSITION is a rule which, by falſe or ſuppoſed numbers, taken at pleaſure, diſcovers the true one required. It is of two kinds, *Single* and *Double*.

SINGLE POSITION,

Is the working with one ſuppoſed number, as if it were the true one, to find the true number.

RULE.

1. Take any number and perform the ſame operations with it as are deſcribed to be performed in the queſtion.
2. Then ſay ; as the ſum of the errors is to the given ſum, ſo is the ſuppoſed number to the true one required.

PROOF. Add the ſeveral parts of the ſum together, and if it agree with the ſum, it is right.

EXAMPLES.

1. Two men, A and B, having found a bag of money, diſputed who ſhould have it ; A ſaid the half third, and one fourth of the money made 130 dollars, and if B could tell how much was in it, he ſhould have it all, otherwiſe he ſhould have nothing ; I demand how much was in the bag ?

OPERATION.

Suppoſe 60 dollars.	As 65 : 130 : : 60
	60
The half 30	
— third 20	65)7800(120 dollars, the anſwer.
— fourth 15	65
65	130
	130
	000

2. A B and C talking of their ages, B ſaid his age was once and a half the age of A ; C ſaid his age was twice and one tenth the age of both and that the ſum of their ages was 93 ; what was the age of each ?
Ans. A's 12, *B*'s 18, *C's* 63 *years.*

3. A perſon having ſpent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had £26$\frac{2}{3}$ left ; what had he at firſt ? *Ans.* £160

4. Seven eighths of a certain number exceeds four fifths by 6 ; what is that number ? *Ans.* 80.

DOUBLE POSITION.

DOUBLE POSITION is that which discovers the true number, or number sought, by making use of two supposed numbers.

RULE.

1. Take only two numbers and proceed with them according to the conditions of the question.
2. Place each error against its respective position or supposed number; if the error be too great, mark it with +; if too small with —
3. Multiply them cross wise, the first position by the last error, and the last position by the first error.
4. If they be alike, that is, both greater or both less than the given number, divide the difference of the products by the difference of the errors, and the quotient will be the answer; but if the errors be unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

EXAMPLES.

1. A man lying at the point of death, left to his three sons all his estate, *viz.* to *F* half wanting 50 dollars; to *G* one third; and to *H* the rest, which was 10 dollars less than the share of *G*. I demand the sum left, and each son's share.

OPERATION.

Suppose the sum 300 dollars.

Then, 300÷2—50=100 *F's* part.
300 ÷ 3=100 *G's* part.
G's part 100—10= 90 *H's* part.

Sum of all their parts 290

Error 10—

Again, Suppose the sum 900 dollars.

Then, 900÷2—50=400 *F's* part.
900÷ 3=300 *G's* part.
G's part 300—10=290 *H's* part.

Sum of all their parts 990

Error 90+

Suppose.	Errors.
300	10—
X	
900	90+
9000	27000
27000	
——— Dollars.	

Having proceeded with the supposed numbers according to the conditions of the question, the *sum of all their parts* must be subtracted from the supposed number; thus the 290 is subtracted from 300, the supposed number, &c.

Sum of the errors. } 100)36000(360 Answer.

The divisor is the sum of the errors 90+and 10—

2. There is a fish whose head is 10 feet long; his tail as long as his head and half the length of his body, and his body as long as his head and tail; what is the whole length of the fish?

Ans. 80 *feet.*

3. A certain man having driven his Swine to market, viz. Hogs, Sows, and Pigs, received for them all 50*l.* being paid for every hog 18*s.* for every ſow 16*s.* for every pig 2*s.* ; there were as many hogs as ſows, and for every ſow there were three pigs ; I demand how many there were of each ſort ?

Ans. 25 *hogs*, 25 *ſows, and* 75 *pigs.*

4. A and B laid out equal ſums of money in trade ; A gained a ſum equal to $\frac{1}{4}$ of his ſtock, and B loſt 225 dollars ; then A's money was double that of B's ; what did each one lay out ? *Ans.* 600 *dollars.*

5. A and B have the ſame income ; A ſaves $\frac{1}{8}$ of his ; but B, by ſpending 30 dollars per annum more than A, at the end of 8 years finds himſelf 40 dollars in debt ; what is their income, and what does each ſpend per annum ?

Ans. their income is 200 *dolls. per ann. A ſpends* 175 *dolls. & B* 205 *per ann.*

§ 11. Discount.

DISCOUNT is an allowance made for the payment of any ſum of money before it becomes due, and is the difference between that ſum, due ſometime hence, and its preſent worth.

The *preſent worth* of any ſum, or debt due ſome time hence, is ſuch a ſum, as, if put to intereſt, would in that time and at the rate *per cent.* for which the diſcount is to be made, amount to the ſum or debt, then due.

RULE.

As the amount of 100 dollars, for the given time and rate is to 100 dollars, ſo is the given ſum to its preſent worth, which ſubtracted from the given ſum, leaves the diſcount.

EXAMPLES.

1. What is the diſcount of Dolls. 321,63 due 4 years hence, at 6 per cent?

OPERATION.

Dolls.
6 intereſt of 100 dolls. 1 year.
4 years.
———
24
100
———
124 amount.

Then, As 124 : 100 : : 321,63
321,63
———
124)32163,00(259,379

321,63 given ſum.
259,379 preſent worth.
———
Ans. 62,251 diſcount.

2. What is the preſent worth of 426 dollars, payable in 4 years and 12 days, diſcounting at the rate of 5 per cent.

Ans. Dolls. 354,515.

§ 12. Equation of Payments.

EQUATION of Payments is the finding of a time to pay at once, ſeveral debts due at different times ſo that neither party ſhall ſuſtain loſs.

RULE.

Multiply each payment by the time at which it is due; then divide the ſum of the products by the ſum of the payments, and the quotient will be the equated time.

EXAMPLES.

1. A owes B 136 dollars, to be paid in 10 months; 69 dollars to be paid in 7 months; and 260 to be paid in 4 months: what is the equated time for the payment of the whole?

OPERATION.

136×	10=1360
96×	7= 672
260×	4=1040
492	3072

492)3072(6 months.
2952

120
30

492)3600(7 days.
3444

156

2. I owe Dolls. 65,125, to be paid $\frac{1}{4}$ in 3 months, $\frac{1}{2}$ in 5 months, $\frac{1}{8}$ in 10 months, and the remainder in 14 months: at what time ought the whole to be paid?

Ans. 6$\frac{1}{4}$ *months.*

3. A merchant has owing to him 300*l.* to be paid as follows, 50*l.* at 2 months, 100*l.* at 5 months, and the rest at 8 months; and it is agreed to make one payment of the whole; I demand when that time must be?

Ans. 6 *months.*

4. A merchant owes me 900 dollars to be paid in 96 days, 130 dollars in 120 days, 500 dollars in 80 days, 1267 dollars in 27 days; what is the mean time for the payment of the whole?

Ans. 63 *days very nearly.*

§ 13. Guaging.

GUAGING is taking the dimenſions of a caſk in inches to find its contents in gallons by the following

METHOD.

1. Add two thirds of the difference between the head and bung diameters to the head diameter for the mean diameter ; but if the ſtaves be but little curving from the head to the bung, add only ſix tenths of this difference.

2. Square the mean diameter, which multiplied by the length of the caſk and the product divided by 294, for wine, or by 359 for ale, the quotient will be the anſwer in gallons.

EXAMPLE.

1. How many ale or beer gallons will a caſk hold, whoſe bung diameter is 31 inches, head diameter 25 inches, and whoſe length is 36 inches ?

OPERATION.

31	Bung diam.	25	head diam
25	Head diam.	4	Two thirds diff.
6	Difference.	29	Mean diam.
		29	
		261	
		58	
		841	Square of mean diam.
		36	Length.
		5046	
		2523	
		359)30276(84 galls. $1\frac{121}{359}$ qts.	

NOTE. 1. In taking the length of the caſk, an allowance muſt be made for the thickneſs for both heads of one inch, 1½ inch, or 2 inches according to the ſize of the caſk.

NOTE. 2. The head diameter muſt be taken cloſe to the chimes, and for ſmall caſks, add 3 tenths of an inch ; for caſks of 40 or 50 gallons, 4 tenths, and for larger caſks, 5 or 6 tenths, and the ſum will be very nearly the head diameter within.

§ 14. Mechanical Powers.

1. OF THE LEVER.

TO find what weight may be raiſed or balanced by any given power, Say, as the diſtance between the body to be raiſed or balanced, and the *fulcrum* or *prop,* is to the diſtance between the prop and the point where the power is applied ; ſo is the power to the weight which it will balance or raiſe.

EXAMPLE.

If a man weighing 150*lb.* reſt on the end of a lever 12 feet long, what weight will he balance on the other end, ſuppoſing the prop $1\frac{1}{2}$ foot from the weight?

12 feet the Lever.
1,5 diſtance of the weight from the fulcrum.

10,5 diſtance from the fulcrum to the man. Therefore,

	Feet.		*Feet.*		*lb.*		*lb.*	
As	1,5	:	10,5	: :	150	:	1050	Ans.

2. OF THE WHEEL AND AXLE.

As the diameter of the axle is to the diameter of the wheel, ſo is the power applied to the wheel, to the weight ſuſpended by the axle.

EXAMPLES.

1. A mechanic wiſhes to make a windlas in ſuch a manner, as that 1 *lb.* applied to the wheel, ſhould be equal to 12 ſuſpended on the axle; now, ſuppoſing the axle 4 inches diameter, required the diameter of the wheel?

	lb.		*in.*		*lb.*		*in.*	
As	1	:	4	: :	12	:	48	Ans. or diameter of the wheel.

2. Suppoſe the diameter of the axle 6 inches and that of the wheel 60 inches, what power at the wheel will balance 10 *lb.* at the axle? *Ans.* 1*lb.*

3. OF THE SCREW.

The power is to the weight to be raiſed as the diſtance between two threads of the ſcrew is to the circumference of a circle deſcribed by the power applied at the end of the lever.

NOTE 1. *To find the circumference of the circle deſcribed by the end of the lever,* multiply the double of the lever by 3,14159, the product will be the circumference.

NOTE 2. It is uſual to abate $\frac{1}{3}$ of the effect of the machine for friction.

EXAMPLES.

There is a ſcrew, whoſe threads are an inch aſunder; the lever by which it is turned is 36 inches long, and the weight to be raiſed a ton, or 2240lb. What power or force muſt be applied to the end of the lever ſufficient to turn the ſcrew, that is, to raiſe the weight?

The lever $36\times2=72\times3{,}14159=226{,}194+$ the circumference.

	circumf.		*in.*		*lb.*		*lb.*
Then, as	226,194	:	1	: :	2240	:	9,903

PROBLEMS.

1. The diameter of a circle being given to find the circumference, multiply the diameter by 3,14159; the product will be the circumference.

2. To find the area of a circle, the diameter being given; multiply the ſquare of the diameter by ,785398; the product is the area.

3. To meaſure the ſolidity of any irregular body whoſe dimenſions cannot be taken, put the body into ſome regular veſſel and fill it with water, then taking out the body, meaſure the fall of water in the veſſel; if the veſſel be ſquare, multiply the ſide by itſelf, and the product by the fall of water, which gives the ſolid contents of the irregular body.

SECTION IV.

xxx:✲:xxx

MISCELLANEOUS QUESTIONS.

IN this Section there is nothing new to be propofed to the fcholar. Enough of Arithmetic has been taught him for all ordinary occurrences in life. It only remains to lead him into fome reflections on the foregoing rules. For this purpofe the following queftions are fubjoined. They are left without *anfwers*, that the fcholar's only refource of knowledge for working them fhould be in his own mind. Mafters having wrought out thefe queftions at a leifure hour, may tranfcribe them with their anfwers into a manufcript for their private ufe, to which on any occafion, without any trouble or hindrance, they may readily advert to fatisfy the inquiries of their pupils.

1. The Northern Lights were firft obferved in London in 1560; how many years fince?
2. What number multiplied by 43 produces 88150?
3. If a cannon may be difcharged twice with 6*lb.* of powder, how many times will 7*C.* 3*qrs.* 17*lbs.* difcharge the fame piece?
4. Reduce 14 guineas and £75 13*s.* 6¾*d.* to Federal Money.
5. What is the intereft of $79,49 one year and five months?
6. A owed B $317,19, for which he gave his note, on intereft, bearing date July 12th, 1797.

On the back of the note are thefe feveral endorfements, viz.
October 17, 1797, Received in cafh, $61,10.
March 20th, 1798, Received 17 *cwt.* of Beef, at $4,33 per *cwt.*
January 1st, 1800, Received in cafh, 84 dollars.
What was there due from A to B, of principal and intereft, Sept. 18th, 1801?

7. What coft 13¼ yards of flannel at 1s. 8½ per yard?
8. What muft I give for 3 *cwt.* 2 *qrs.* 13*lb.* of cheefe at 7 cents per lb.?
9. What will 35 yards of broadcloth coft at 23*s.* 6*d.* per yard?
10. What will be the coft of a line of veal, weighing 16½*lb.* at 2½*d.* per bl.?
11. What will 87¼*lb.* of tallow coft, at 9¼*d.* per *lb.*?
12. What will 196 yards of tape coft, at 3 farthings per yard?
13. What will 56 bufhels of oats coft, at 2*s.* 3½*d.* per bufhel?
14. At £3 7*s.* 6*d.* per *cwt.* for fugar, what is that per lb.?
15. How much in length of a board that is 10 inches wide will it require to make a fquare foot?
16. How many fquare feet in a board 1 foot, 3 inches wide, and 14 feet, 9 inches long?
17. How much wood in a load 9 feet long, 3½ feet wide, and 2 feet 9 inches high?
18. At $1,33 per yard for cloth, what muft I give for 72 yards?
19. If 2¼ *cwt.* of cotton wool coft £11 17*s.* 6*d.* what is that per *lb.*?
20. If 1832½ gallons of wine coft £44 6*s.* what is that per gallon?
21. What will 53½*lb.* of beef coft at 5 cents 5 mills per *lb.*?
22. What will 50 bufhels of potatoes coft at 21 cents per bufhel?

23. At $10,76 per *cwt.* for ſugar, what is that per *lb.* ?

24. What will be a man's wages for 6 months, at 43 cents per day, working $5\frac{1}{2}$ days per week ?

25. What muſt I give for paſturing my horſe 19 weeks, at 33 cents per week ?

26. How many revolutions does the moon perform in 144 years, 2 days, 10 hours ; one revolution being in 27 days, 7 h. 43 m. ?

27. What will 7 pieces of cloth, containing 27 yards each, come to, at 15*s.* $4\frac{1}{2}$*d.* per yard ?

28. A man ſpends 23 dollars 69 cents, 5 mills, in a year, what is that per day ?

29. Suppoſe the Legiſlature of this State ſhould grant a tax of 7 cents 3 mills on a dollar, what will a man's tax be, who is 142 dollars 40 cents on the liſt ?

30. A Bankrupt, whoſe effects are 3948 dollars, can pay his creditors but 28 cents 5 mills on the dollar ; what does he owe ?

31. Suppoſe a ciſtern having a pipe that conveys 4 gallons, 2 qts. into it in an hour, has another that lets out 2 gallons, 1 qt. 1 pt. in an hour ; if the ciſtern contains 84 gallons, in what time will it be filled ?

32. If 80 dollars worth of proviſions will ſerve 20 men 25 days, what number of men will the ſame proviſions ſerve 10 days ?

33. If 6 men ſpend 16 dollars 7 cents, in 40 days ; how long will 135 men be in ſpending 100 dollars ?

34. A bridge built acroſs a river in 6 months, by 45 men, was waſhed away by the current ; required the number of workmen ſufficient to build another of twice as much worth in 4 months ?

35. Four men, A, B, C, and D found a purſe of money containing 12 dollars, they agree that A ſhall have one third, B one fourth, C one ſixth, and D one eighth of it ; what muſt each one have according to this agreement ?

36. A certain uſurer lent 90*l.* for twelve months, and received principal and intereſt 95*l.* 8*s.* I demand at what rate per cent. he received intereſt ?

37. If a gentleman have an eſtate of 1000*l.* per ann. how much may he ſpend per day to lay up three ſcore guineas at the year's end ?

38. What is the length of a road, which being 33 feet wide contains an acre ?

39. Required a number from which if 7 be ſubtracted and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 added to the product, the ſquare root of the ſum extracted, and three fourths of that root cubed, the cube divided by 9, the laſt quotient may be 24 ?

40. If a quarter of wheat affords 60 ten penny loaves, how many eight penny loaves may be obtained from it ?

41. If the carriage of 7 cwt. 2qr. for 105 miles be 1*l.* 5*s.* how far may 5 cwt. 1 qr. be carried for the ſame money ?

42. If 50 men conſume 15 buſhels of grain in 40 days, how much will 30 men conſume in 60 days ?

43. On the ſame ſuppoſition, how long will 50 buſhels maintain 64 men ?

44. A gentleman having 50*s.* to pay among his laborers for a day's work, would give to every boy 6d. to every woman 8d. and to every man 16d. the number of boys, women and men, was the ſame, I demand the number of each ?

45. A gentleman had 7*l.* 17*s.* 6*d.* to pay among his laborers ; to every boy he gave 6*d.* to every woman 8*d.* and to every man 16*d.* and there were for every boy three women, and for every woman two men ; I demand the number of each ?

46. Three Gardeners, A, B, and C, having bought a piece of ground, find the profits of it amount to 120*l.* per annum. Now the ſum of money which they laid down was in ſuch proportion, that as often as A paid 5*l.* B paid 7*l.* and as often as B paid 4*l.* C paid 6*l.* I demand how much each man muſt have per annum of the gain ?

47. A young man received 210*l.* which was $\frac{2}{3}$ of his eldeſt brother's portion : now three times the eldeſt brother's portion was half the father's eſtate ; I demand how much the eſtate was ?

48. Two men depart both from one place, the one goes North and the other South ; the one goes 7 miles a day, the other 11 miles a day ; how far are they diſtant the 12th day after their departure ?

49. Two men depart both from one place and both go the ſame road ; the one travels 12 miles every day, the other 17 miles every day ; how far are they diſtant the 10th day after their departure ?

50. The river Po is 1000 feet broad, and 10 feet deep, and it runs at the rate of 4 miles an hour. In what time will it diſcharge a cubic mile of water (reckoning 5000 feet to the mile) into the ſea ?

51. If the country which ſupplies the river Po with water be 380 miles long and 120 broad, and the whole land upon the ſurface of the earth be 62,700,000 ſquare miles, and if the quantity of water diſcharged by the rivers into the ſea be every where proportional to the extent of land by which the rivers are ſupplied ; how many times greater than the Po will the whole amount of the rivers be ?

52. Upon the ſame ſuppoſition, what quantity of water altogether will be diſcharged by all the rivers into the ſea in a year ?

53. If the proportion of the ſea on the ſurface of the earth to that of land be as $10\frac{1}{2}$ to 5, and the mean depth of the ſea be a quarter of a mile ; how many years would it take if the ocean were empty to fill it by the rivers running at the preſent rate ?

54. If a cubic foot of water weigh 1000 oz. avoirdupois, and the weight of mercury be $13\frac{1}{2}$ times greater than of water, and the height of the mercury in the barometer (the weight of which is equal to the weight of a column of air on the ſame baſe, extending to the top of the atmoſphere) be 30 inches ; what will be the weight of the air upon a ſquare foot ? a ſquare mile ? and what will be the whole weight of the atmoſphere, ſuppoſing the ſize of the earth as in queſtions 51 and 53 ?

55. A began trade June 1, with 40 dollars, and took in B as a partner, Sept. 8, following, with 120 dollars ; on Dec. 24, A put in 190 dollars more, and continued the whole in trade till May 5, following, when their whole gain was found to be 82 dollars ; what is each partner's ſhare ?

56. If I give 80 buſhels of potatoes, at 21 cents per buſhel, and 240*lb.* of flax, at 15 cents per lb. for 64 buſhels of ſalt, what is the ſalt per buſhel ?

57. What is the preſent worth of 482 dollars, payable 4 years hence, diſcounting at the rate of 6 per cent ?

58. I have owing to me as follows, viz. \$18,73 in 8 months ; \$46,00 in 5 months ; and \$104,84 in 3 months ; what is the mean time for the payment of the whole ?

59. If I ſell 500 deals at 15*d.* a piece, and loſe £9 per cent. what do I loſe in the whole quantity ?

60. If I buy 1000 Ells Flemiſh of linen for £90, what may I ſell it per Ell in London, to gain £10, in the whole ?

Pleaſant and Diverting Queſtions.

1. THERE was a well 30 feet deep; a Frog at the bottom could jump up 3 feet every day, but he would fall back two feet every night. How many days did it take the Frog to jump out?

2. Two men were driving ſheep to market, ſays one to the other, give me one of yours and I ſhall have as many as you; the other ſays, give me one of yours and I ſhall have as many again as you. How many had each?

3. As I was a going to St. Ives,
I met ſeven wives,
Every wife had ſeven ſacks,
Every ſack had ſeven cats,
Every cat had ſeven kits,
Kits, cats, ſacks and wives,
How many were going to St. Ives?

4. The account of a certain ſchool is as follows, viz. $\frac{1}{16}$ of the boys learn geometry, $\frac{3}{8}$ learn grammar, $\frac{3}{16}$ learn arithmetic, $\frac{3}{16}$ learn to write, and 9 learn to read; I demand the number of each?

5 A man driving his geeſe to market, was met by another, who ſaid, Good-morrow, maſter, with your hundred geeſe; ſays he I have not an hundred, but if I had half as many as I now have, and two geeſe and an half beſide the number I now have already, I ſhould have an hundred: How many had he?

6. Three travellers met at a caravanſary, or inn, in Perſia; and two of them brought their proviſions along with them, according to the cuſtom of the country; but the third not having provided any, propoſed to the others that they ſhould eat together, and he would pay the value of his proportion. This being agreed to, A produces 5 loaves, and B 3 loaves, which the travellers eat together, and C paid 8 pieces of money as the value of his ſhare, with which the others were ſatisfied, but quarrelled about the dividing of it. Upon this the affair was referred to the judge, who decided the diſpute by an impartial ſentence. Required his deciſion?

7. Suppoſe the 9 Digits to be placed in a quadrangular form; I demand in what order they muſt ſtand, that any three figures in a right line, may make juſt 15.

8. A countryman having a Fox, a Gooſe, and a peck of Corn, in his journey came to a river, where it ſo happened that he could carry but one over at a time. Now as no two were to be left together that might deſtroy each other; ſo he was at his wits end how to diſpoſe of them: for, ſays he, the corn can't eat the gooſe, nor the gooſe eat the fox, yet the fox can eat the gooſe, and the gooſe eat the corn. The queſtion is how he muſt carry them over, that they might not devour each other.

9. Three jealous huſbands with their wives, being ready to paſs by night over a river, do find at water ſide a boat which can carry but two perſons at once, and for want of a waterman they are neceſſitated to row themſelves over the river at ſeveral times: The queſtion is, how thoſe 6 perſons ſhall paſs by 2 and 2, ſo that none of the three wives may be found in the company of one or two men, unleſs her huſband be preſent?

10. Two merry companions are to have equal ſhares of 8 gallons of wine, which are in a veſſel containing exactly 8 gallons; now to divide it equally between them, they have only two other empty veſſels, of which one contains 5 gallons, and the other 3; the queſtion is, how they ſhall divide the ſaid wine between them by the help of theſe three veſſels, ſo that they may have 4 gallons a piece?

SECTION III.

Forms of Notes, Deeds, Bonds, and other Inſtruments of Writing.

§. 1. OF NOTES.

No. I.

Overdean, Sept. 17, 1802. For value received I promiſe to pay to Oliver Bountiful, or order, ſixty-three dollars, fifty-four cents, on demand, with intereſt after three months. *William Truſty.*

Atteſt, *Timothy Teſtimony.*

No. II.

Bilfort, Sept. 17, 1802. For value received, I promiſe to pay O. R. or bearer———dollars———cents, three months after date.

Peter Pencil.

No. III.

BY TWO PERSONS.

Arian, Sept. 17, 1802. For value received we jointly and ſeverally promiſe to pay C. D. or order, —— dollars —— cents on demand, with intereſt.

Atteſt, *Alden Faithful.*

Conſtance Adley. *James Fairface.*

OBSERVATIONS.

1. No note is negotiable unleſs the words, *or order,* otherwiſe, *or bearer,* be inſerted in it.

2. If the note be written to pay him "*or order,*" (No. 1.) then *Oliver Bountiful* may endorſe this note, that is write his name on the back ſide and ſell it to A, B, C, or whom he pleaſes. Then A, who buys the note, calls on *William Truſty* for payment, and if he neglects or is unable to pay, A may recover it of the *endorſer.*

3. If a note be written, to pay him "*or bearer,*" (No 2.) then any perſon who holds the note may ſue and recover the ſame of Peter Pencil.

4. The rate of intereſt eſtabliſhed by law being *ſix per cent. per annum,* it becomes unneceſſary, in writing notes to mention the rate of intereſt; it is ſufficient to write them for the payment of ſuch a ſum, with intereſt, for it will be underſtood, legal intereſt, which is *ſix per cent.*

5. All notes are either payable on demand, or at the expiration of a certain term of time agreed upon by the parties and mentioned in the note, as three months, or a year, &c.

6. If a bond or note mention no time of payment, it is always on demand, whether the words "*on demand*" be expreſſed or not.

7. All notes payable at a certain time are on interest as soon as they become due, though in such notes there be no mention made of interest.

This rule is founded on the principle, that every man ought to receive his money when due, and that the non payment of it at that time is an injury to him. The law, therefore, to do him justice, allows him interest from the time the money becomes due, as a compensation for the injury.

8. Upon the same principle a note payable on demand, without any mention made of interest is on interest after a demand of payment, for upon demand such notes immediately become due.

9. If a note be given for a specific article, as rye, payable in one, two or three months, or in any certain time, and the signer of such note suffers the time to elapse without delivering such article, the holder of the note will not be obliged to take the article afterwards, but may demand and recover the value of it in money.

§ 2. OF BONDS.

A Bond, with a condition from one to another.

KNOW all men by these presents, that I, C. D. of, &c. in the county of &c. am held and firmly bound to E. F. of, &c. in two hundred dollars to be paid to the said E. F. or his certain attorney, his executors, administrators or assigns; to which payment, well and truly to be made, I bind myself, my heirs, executors, and administrators, firmly by these presents: Sealed with my seal. Dated the eleventh day of——in the year of our Lord one thousand eight hundred and two.

The condition of this obligation is such, That if the above bound C. D. his heirs, executors or administrators, do and shall well and truly pay or cause to be paid, unto the above named E. F. his executors, administrators or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of——next ensuing the date hereof: Then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

A Condition of a Counter Bond, or Bond of Indemnity, where one man becomes bound for another.

THE condition of this obligation is such, That whereas the above named A. B. at the special instance and request, and for the only proper debt of the above bound C. D. together with the said C. D. is, in and by one bond or obligation bearing equal date with the obligation above written, held and firmly bound unto E. F. of, &c. in the penal sum of————dollars, conditioned for the payment of the sum of, &c. with legal interest for the same, on the——day of——next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written may more fully appear: If therefore the said C. D. his heirs, executors or administrators, do and shall well and truly pay or cause to be paid unto the said E. F. his executors, administrators, or assigns, the said sum of, &c. with legal interest for the same, on the said——day of, &c. next ensuing the date of the said in part recited obligation, according to the true intent and meaning, and in full discharge and satisfaction of the said in part recited bond or obligation: Then, &c. Otherwise, &c.

Note. The principal difference between a note and a bond is that the latter is an inftrument of more folemnity, being given under feal. Alfo, a note may be controled by a fpecial agreement, different from the note, whereas, in cafe of a bond, no fpecial agreement can in the leaft controul what appears to have been the intention of the parties as expreffed by the words in the condition of the bond.

§ 3. OF RECEIPTS.

No. I.

Sitgrieves, Sept. 19, 1802. Received from Mr. Durance Adley, ten dollars in full of all accounts.

Orvand Conftance.

No. II.

Sitgrieves, Sept. 19, 1802. Received from Mr. Orvand Conftance, five dollars in full of all accounts.

Durance Adley.

No. III.

A Receipt for an endorfement on a Note.

Sitgrieves, Sept. 19, 1802. Received from Mr. Simpfon Eafley, (by the hand of Mr. Titus Trufty,) fixteen dollars twenty five cents, which is endorfed on his note of June 3, 1802.

Peter Cheerful.

No. IV.

A Receipt for money received on Account.

Sitgrieves, Sept. 19, 1802. Received of Mr. Orand Landlike, fifty dollars on account.

Eldro Slackley.

No. V.

A Receipt for intereft due on a Bond.

Received this——day of——of Mr. A. B. the fum of five pounds, in full of one year's intereft of 100l. due to me on the——day of——laft, on bond from the faid A. B. I fay received. By me, C. D.

OBSERVATIONS.

1. There is a diftinction between receipts given in full of *all accounts,* and others in full of *all demands.* The former cut off accounts *only;* the latter cut off not only accounts, but all obligations and right of action.

2. When any two perfons make a fettlement and pafs receipts (No. I. and No. II.) each receipt muft fpecify a particular fum, received, lefs or more. It is not neceffary that the fum fpecified in the receipt, be the exact fum received.

§ 4. OF ORDERS.

No. I.

Mr. Stephen Burgess.

SIR,

For value received, pay to A. B. Ten Dollars, and place the ſame to my account. *Samuel Skinner.*

Archdale, Sept. 9, 1802.

No. II.

Boſton, Sept. 9, 1802.

SIR,

For value received, pay G. R. eighty ſix cents, and this, with his receipt, ſhall be your diſcharge from me.

Nicholas Reubens.

To Mr. James Robottom.

§ 5. OF DEEDS.

No. I.

A Warranty Deed.

KNOW ALL MEN BY THESE PRESENTS, That I, Peter Careful, of Leominſter, in the County of Worceſter and Commonwealth of Maſſachuſetts, gentleman, for and in conſideration of one hundred fifty dollars, and forty five cents paid to me by Samuel Pendleton, of Aſhby, in the County of Middleſex, and Commonwealth of Maſſachuſetts, yeoman, the receipt whereof I do hereby acknowledge, do hereby give, grant, ſell and convey to the ſaid Samuel Pendleton, his heirs and aſſigns, a certain tract and parcel of land, bounded as follows, viz.

[*Here inſert the bounds, together with all the privileges and appurtenances thereunto belonging.*]

To have and to hold the ſame unto the ſaid Samuel Pendleton, his heirs and aſſigns to his and their uſe and behoof forever. And I do covenant with the ſaid Peter Pendleton, his heirs and aſſigns, That I am lawfully ſeized in fee of the premiſes, that they are free of all incumbrances, and that I will warrant and defend the ſame to the ſaid Peter Pendleton, his heirs and aſſigns forever, againſt the lawful claims and demands of all perſons.

In witneſs whereof I hereunto ſet my hand and ſeal, this——day of—— in the year of our Lord one thouſand eight hundred and two.

Signed, ſealed and delivered in preſence of } *Peter Careful.* O

L. R.
F. G.

No. II.

Quitclaim Deed.

KNOW ALL MEN BY THESE PRESENTS, That I, A. B. of, &c. in conſideration of the ſum of——to be paid by C. D. of, &c. the receipt whereof I do hereby acknowledge, have remiſſed, releaſed, and forever quit-claimed, and do by theſe preſents remiſs, releaſe, and forever quit-claim unto the ſaid C. D. his heirs and aſſigns forever. (*Here inſert the premiſes.*) To have and to hold the ſame, together with all the privileges and appurtenances thereunto belonging, to him the ſaid C. D. his heirs and aſſigns forever.—*In witneſs, &c.*